

Vertical Orbit-Excursion FFAGs (VFFAGs)

a.k.a. FFAG Cyclotrons (Ohkawa, 1955) a.k.a. Helicoidal FFAGs (Leleux, 1959) a.k.a. Ring Cyclotrons (Teichmann, 1960-2)

- I. Principle & Magnetic Fields
- II. Proton Driver Study
- III. Isochronous Machines
- IV. Three-Lens Horizontal FFAGs
- V. Proton Omni-Ring

Vertical Orbit-Excursion FFAGs (VFFAGs) for applications

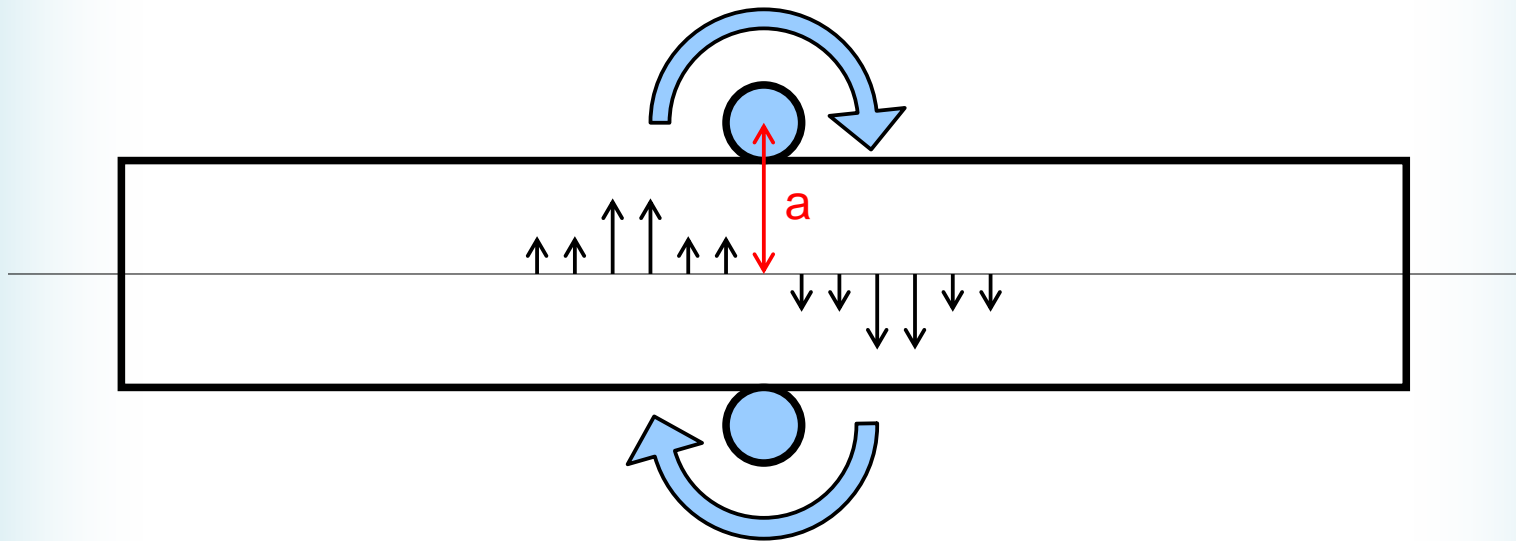
a.k.a. computers and superconducting magnets weren't much good in 1960 but now they are

- I. Smaller Hadron Therapy Magnets
- II. Neutrino Factory, ISIS Upgrade
- III. Transmutation, Antimatter(?)
- IV. Design by Artificial Intelligence
- V. High Beam Dynamics R&D per £

I. Principle & Magnetic Fields

Horizontal SC magnet problem

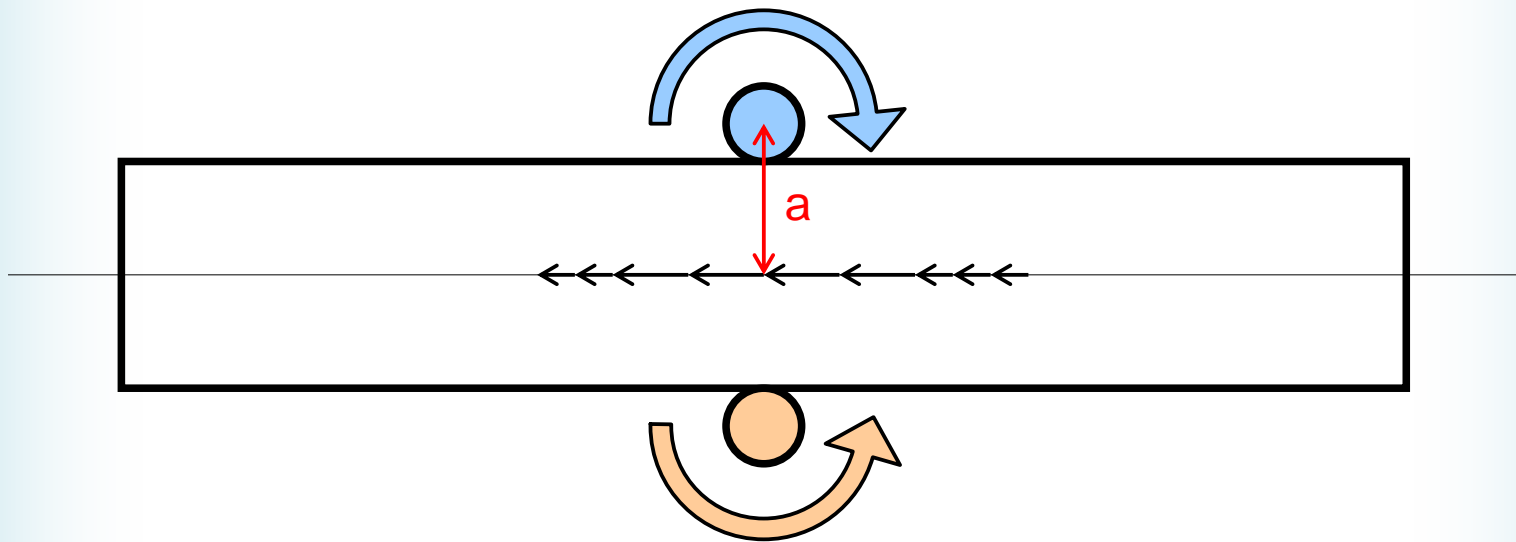
- Getting vertical B field requires same-direction current windings (nearby)



- B_y proportional to $x/(a^2+x^2)$

Horizontal SC magnet variation

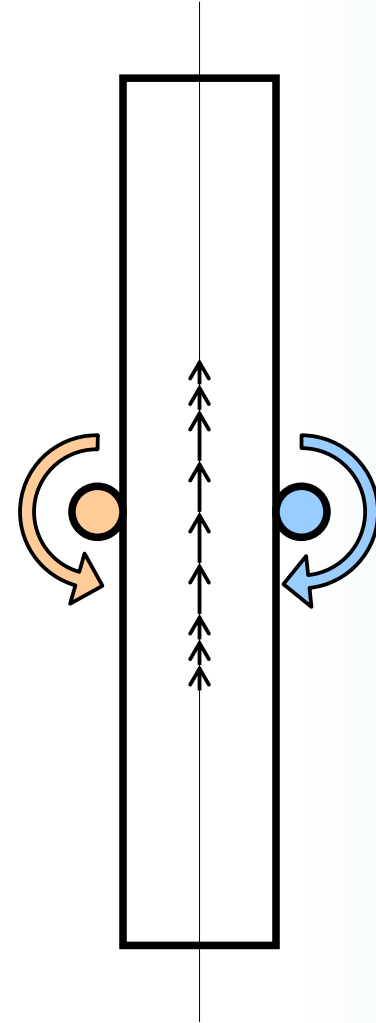
- Getting horizontal B field requires opposite current windings and is easier



- B_x proportional to $a/(a^2+x^2)$

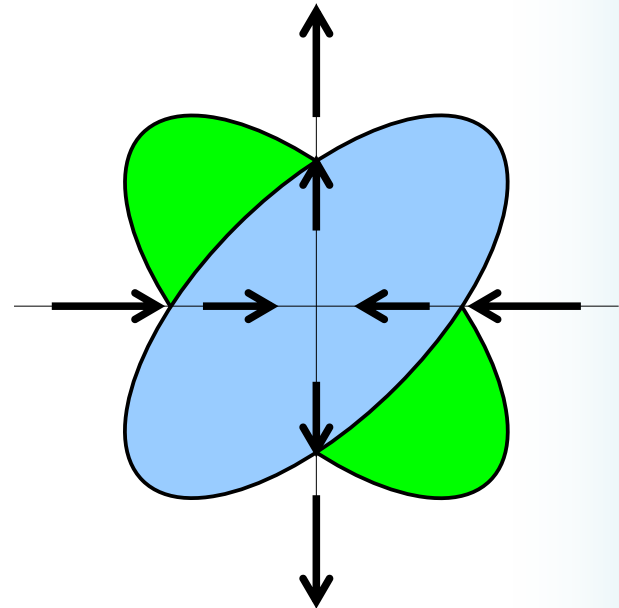
Vertical SC magnet

- But now the field is in the wrong direction!
- That's OK, rotate the magnet
- The dipole field is there
- But what sort of focussing does this magnet give?



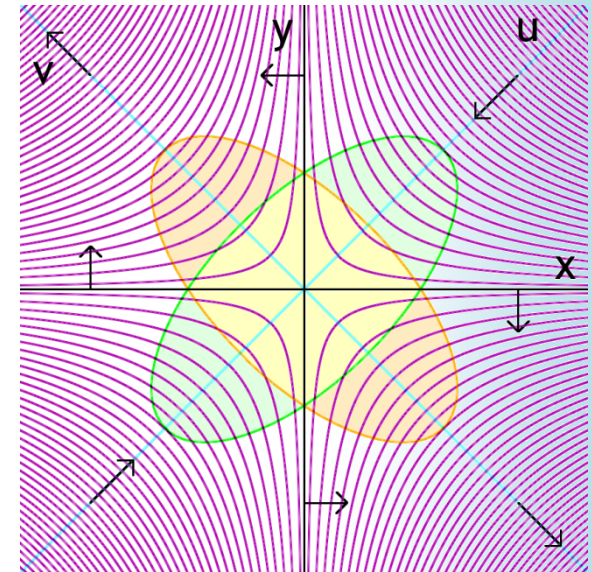
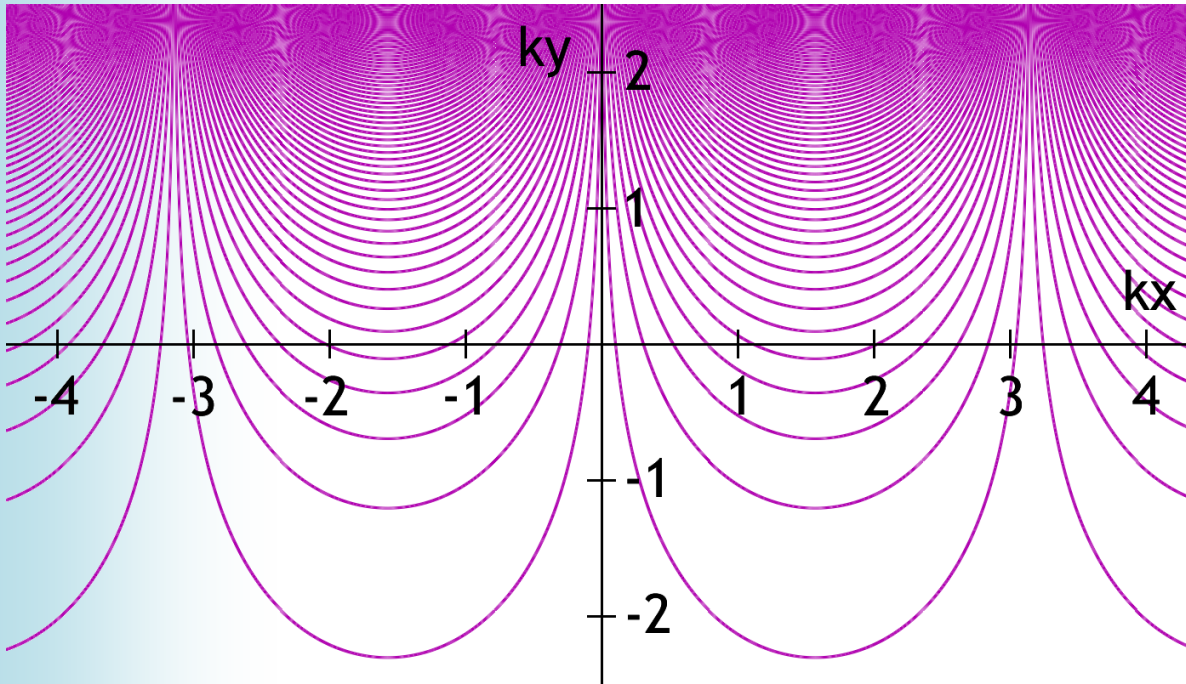
“Scaling” VFFAG magnet

- Dipole field should increase moving up the magnet, so set $B_y = B_0 e^{ky}$ on axis ($x=0$)
- Subtracting dipole component leaves the field of a skew quad:
 - Exponential is good because moving upwards just scales the field and all gradients
 - Thus closed orbits at different momenta are exactly the same shape, just translated upwards
 - VFFAG = Vertical orbit excursion FFAG



Scaling VFFAG Field & Scaling Law

$$B_y = B_0 e^{ky} \cos kx \quad B_x = -B_0 e^{ky} \sin kx$$



$$y \mapsto y + \Delta y, \quad (p, \mathbf{B}) \mapsto (p, \mathbf{B}) e^{k\Delta y}$$

FODO Scaling VFFAG Machine

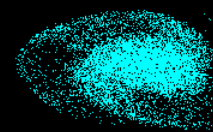
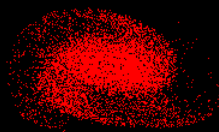
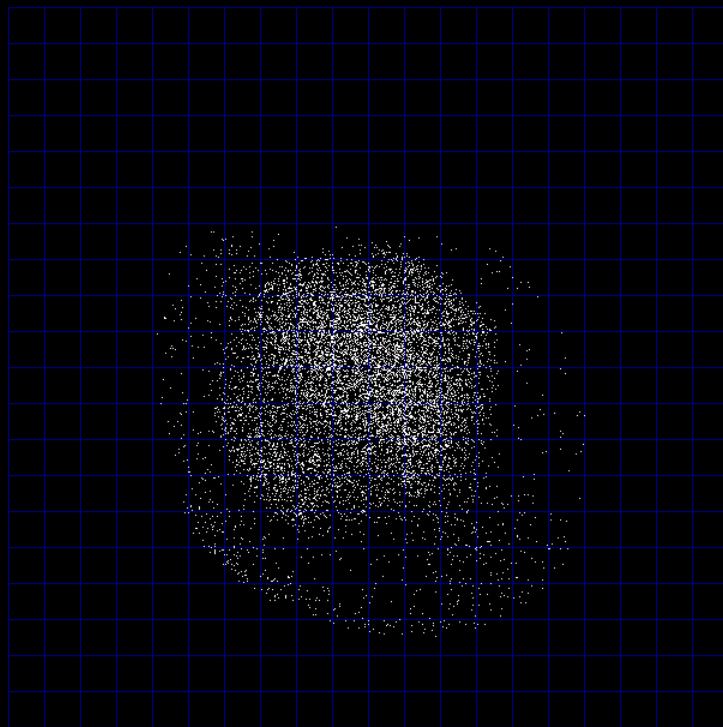
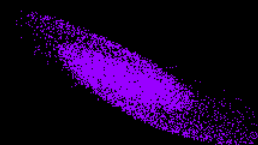
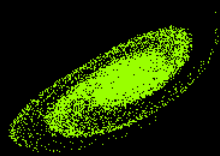
- First VFFAG tracking simulation, for HB2010
 - 2D, zero space charge, nonlinear magnets
- 150mm.mrad
 ϵ_{geom} input beam
- Proton-driver-like
but nasty
circumference
factor! (C=17)

$$C = \langle |\mathbf{B}| \rangle / \langle B_y \rangle$$

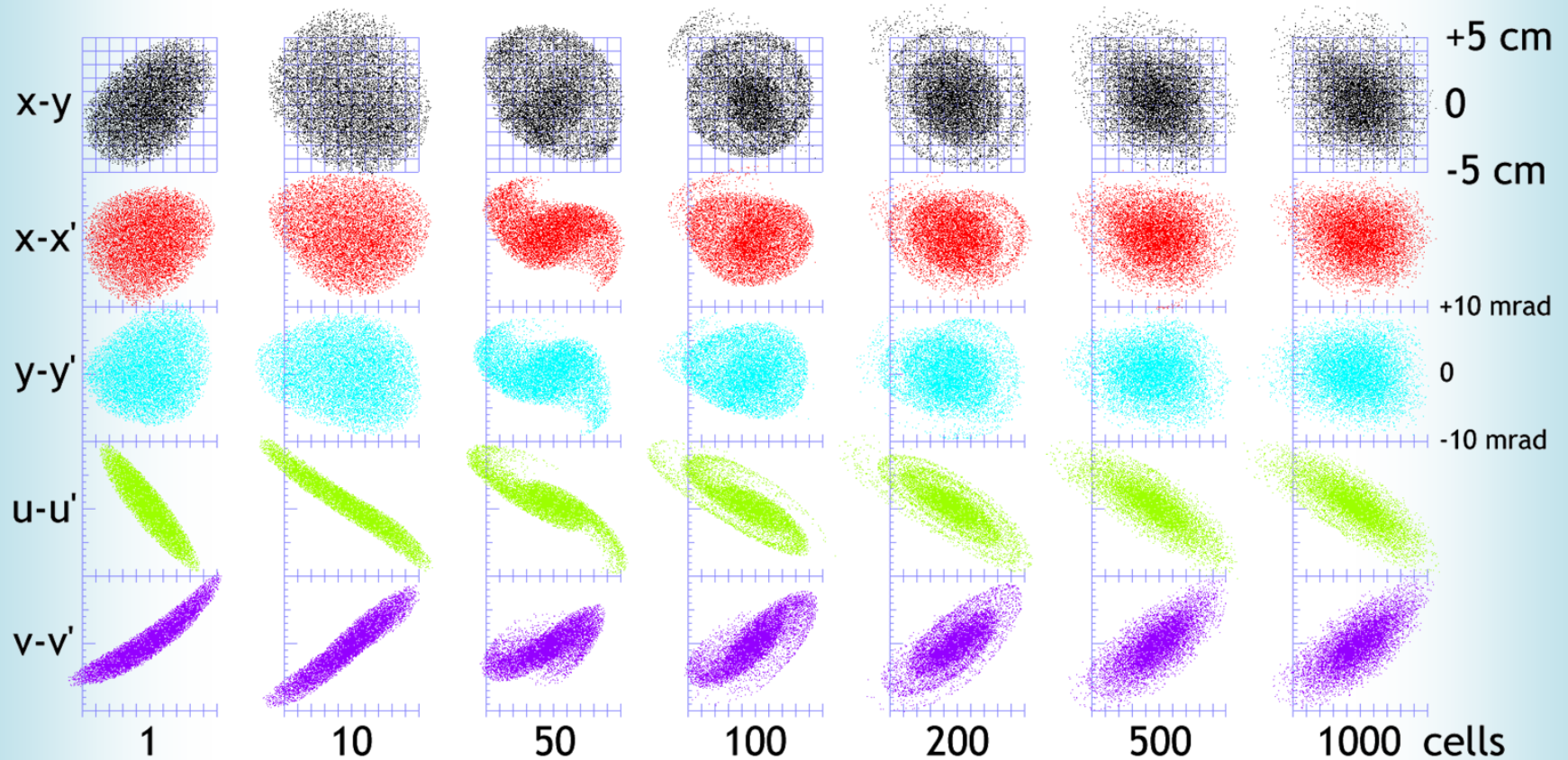
Table 1: Parameters of the FODO lattice.

Energy range	800 MeV–12 GeV
Orbit excursion	43.5 cm (vertical)
k	5 m^{-1}
B_0	0.5 T
B_{max}	4.41 T (beam centre) 4.96 T (beam top) 5.33 T (whole magnet)
Lattice	FODO
F length	0.4 m
D length	0.45 m
Drift length	4 m

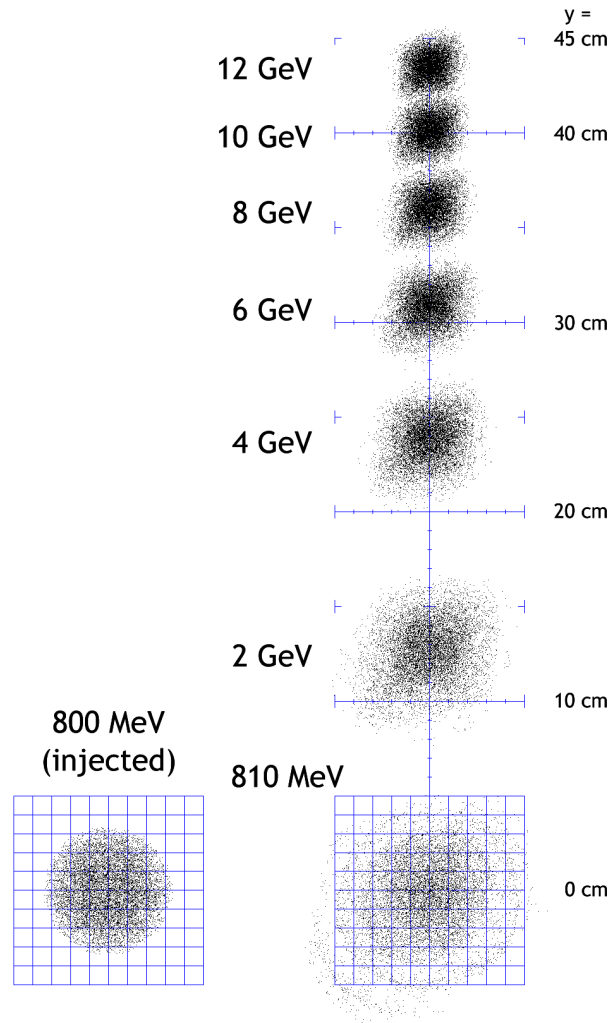
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time=0.00405439ms
beam=100%



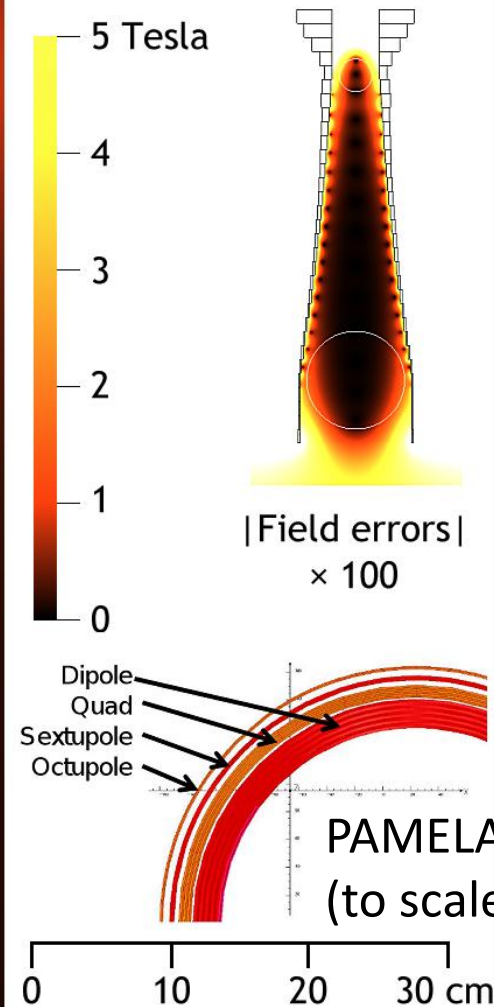
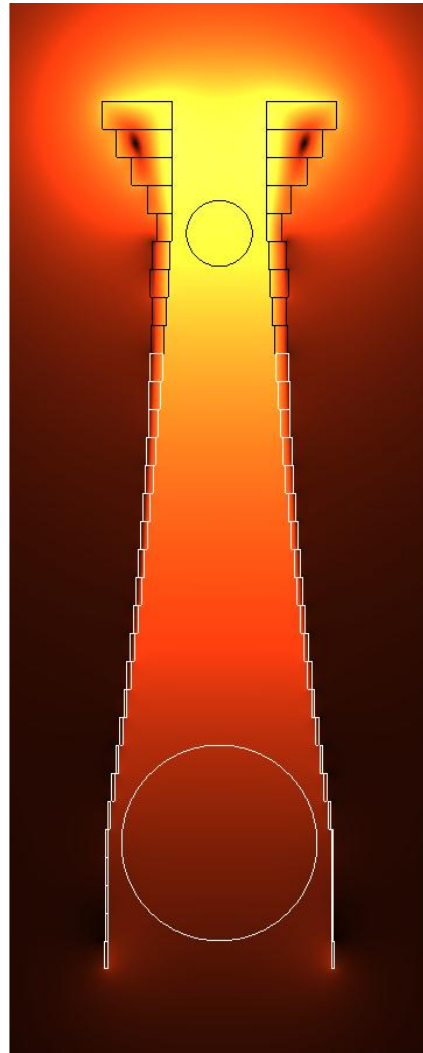
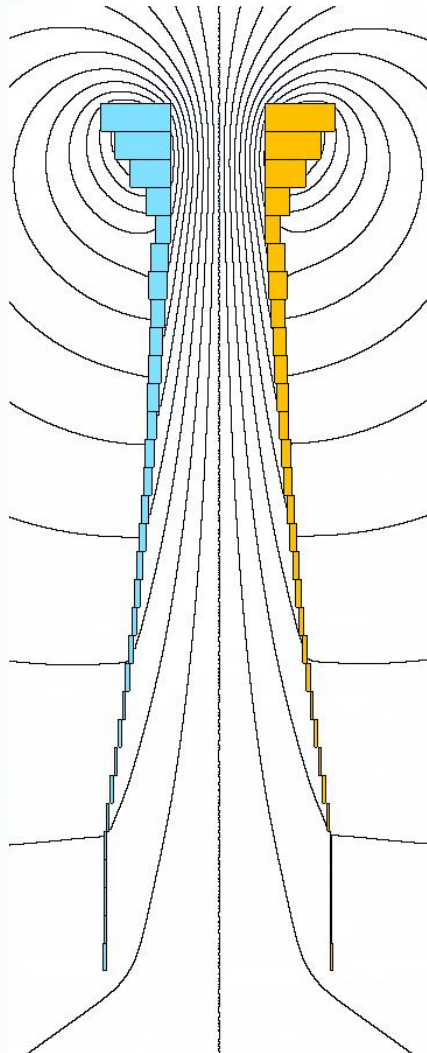
Scaling VFFAG Tracking



VFFAG Acceleration



2D Winding Model for Magnet



Application: Hadron Therapy?

- Low intensity but high rep-rate
 - Fixed field is a plus, space charge not too bad
- Small beams
 - The VFFAG magnet can be a narrow vertical slot
 - Less stored energy, smaller windings required
- Fixed tune allows slower acceleration, less RF
- Disadvantage: we still have the FFAG extraction-from-an-orbit-that-moves problem

II. Proton Driver Study

Motivation: ISIS Energy Booster

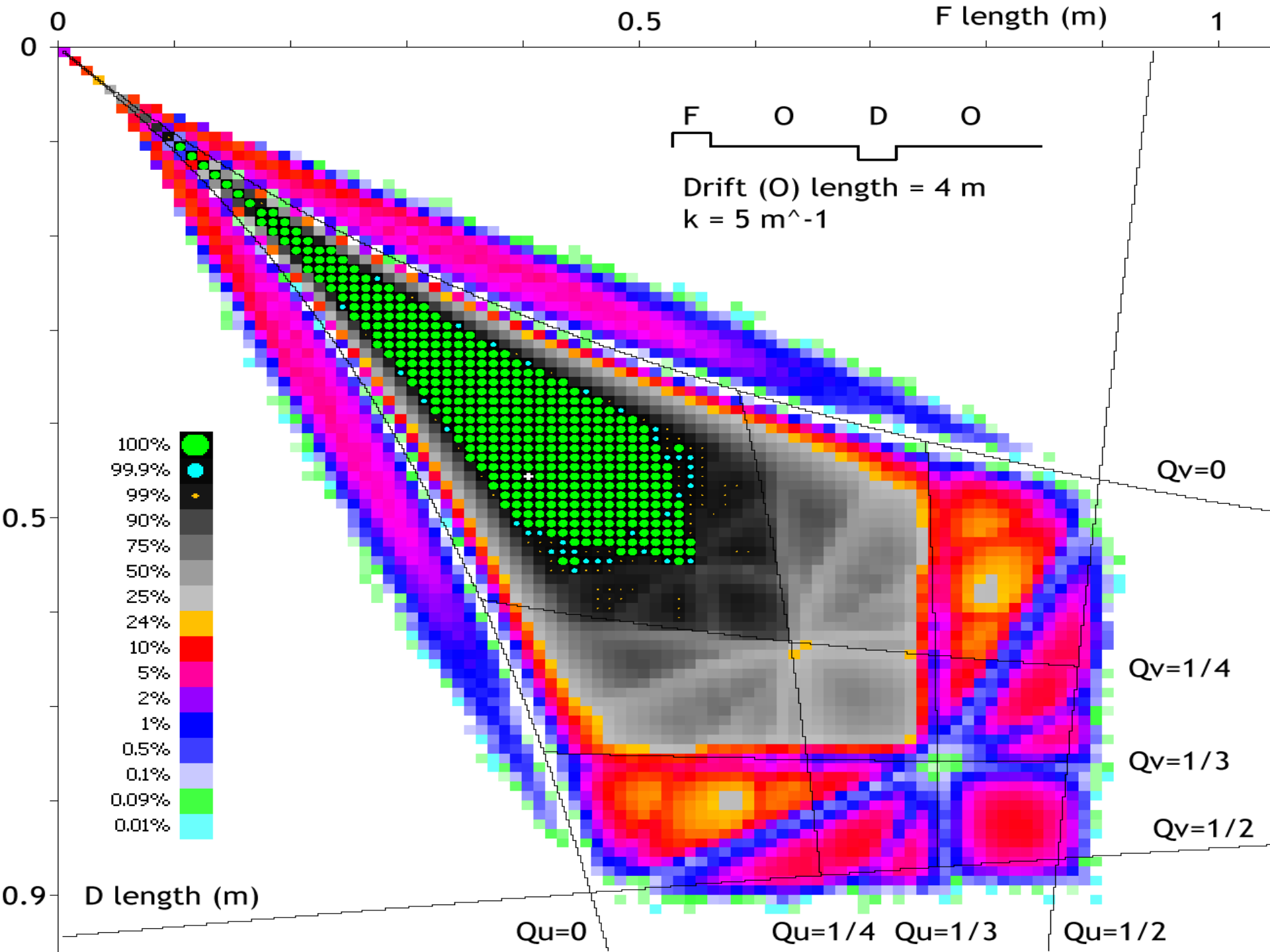
- FFAG of some sort (but with 2-4m drifts)
- Superconducting magnets
- Energy: 800MeV – 12GeV
- Ring radius 52m (2x ISIS) could do 2.5x,3x
- Mean dipole field in magnets 0.47 – 4.14T
- 30% RF packing factor, 20% magnets
- Warm 6.2 – 7.3MHz RF
- Harmonic number 8 (10,12 in larger ring)

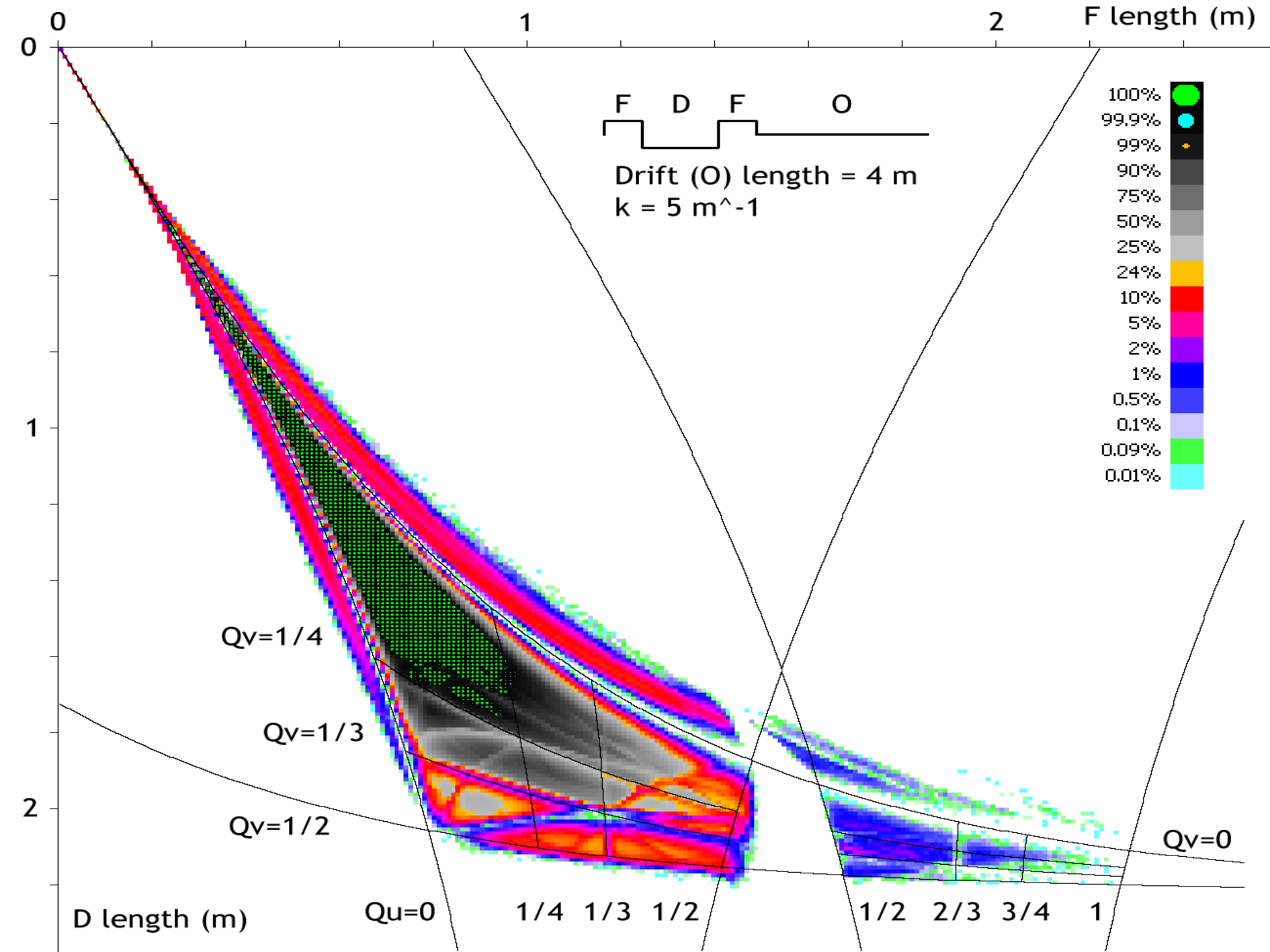
Why 12GeV? (= 2.5MW at 208uA)

- Existing 2RF is 2x11kV in 1.9707m module
 - $11.16\text{kV/m} * 20\text{ms} * c = 67\text{GeV}$
- Assume 30% ring RF packing factor
 - $67\text{GeV} * 30\% = 20\text{GeV}$
- Assume $\langle \cos \phi \rangle = 0.7$ ($\phi \sim 45^\circ$)
 - $20\text{GeV} * 0.7 = 14\text{GeV}$
- Finally, velocity goes from 0.84c to $\sim 0.99c$
 - $14\text{GeV} * 0.9 = 12.6\text{GeV}$

Scaling (V)FFAG disease

- Defocussing is locked to reverse bending, as in scaling FFAGs → large circumference factor
- Searched for “lopsided” scaling lattices with good dynamic aperture [HB2010]
 - 10000 particles were tracked for 1km
 - Survival rate plotted on axes of lengths of “F” and “D” type magnets
 - This reveals both the lattice stability region and resonance stop-bands

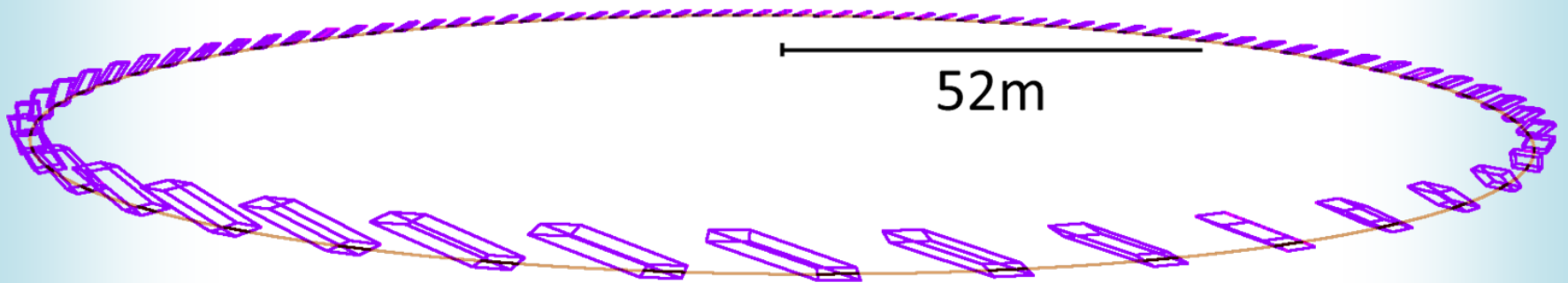




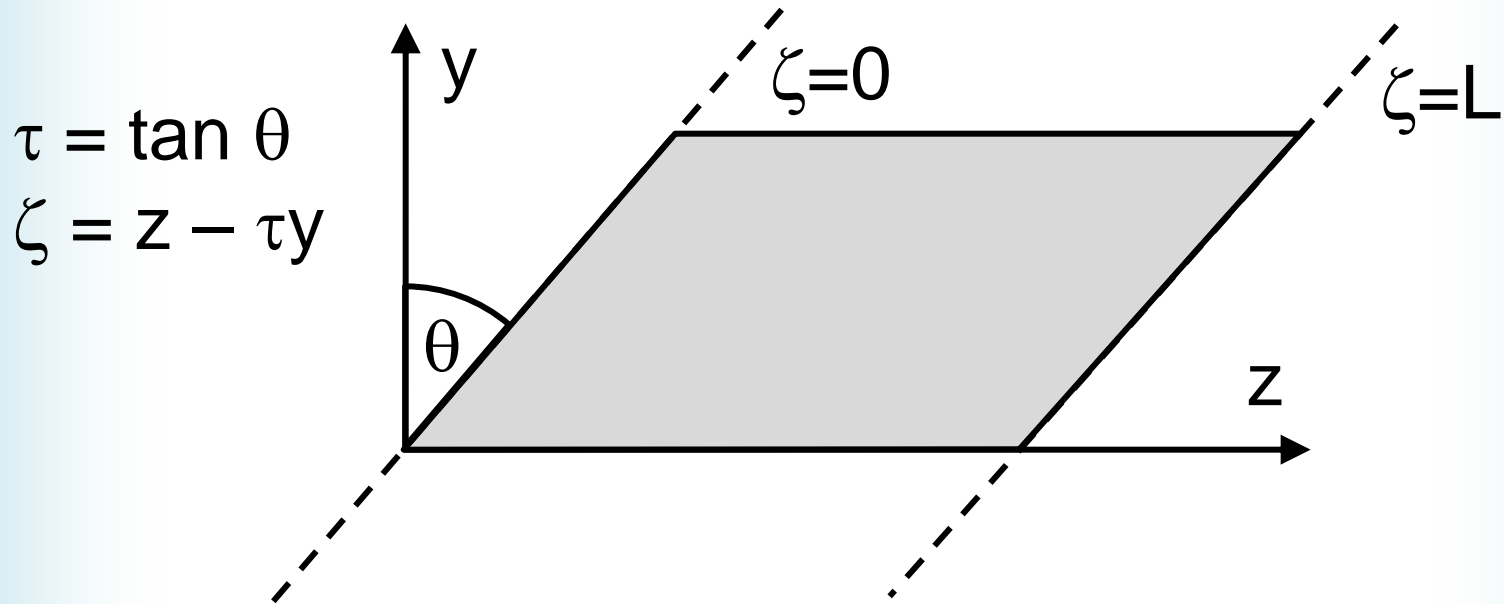
Lattices can't be very lopsided

- Unfortunately in all cases the region of dynamic stability sticks very close to the $F=D$ diagonal line
- The 2nd FDF stability region as used in PAMELA does not have enough dynamic aperture
- So basic scaling VFFAGs will always be big, with much reverse bending
 - Could edge focussing avoid reverse bends?

[HB2012]



VFFAG with Edge Focussing



one wants a mid-plane field $B_y = B_0 e^{ky} f(\zeta)$ but to obey Maxwell's equation $(\nabla \times \mathbf{B})_x = 0$, this has to be modified to $(B_y, B_z) = B_0 e^{ky} (f(\zeta) - \frac{\tau}{k} f'(\zeta), \frac{1}{k} f'(\zeta))$.

Scaling law: $y \mapsto y + \Delta y, \quad (p, \mathbf{B}) \mapsto (p, \mathbf{B}) e^{k\Delta y}$

$$z \mapsto z + \tau \Delta y$$

Spiral Scaling VFFAG Magnet Field

- 5GeV design

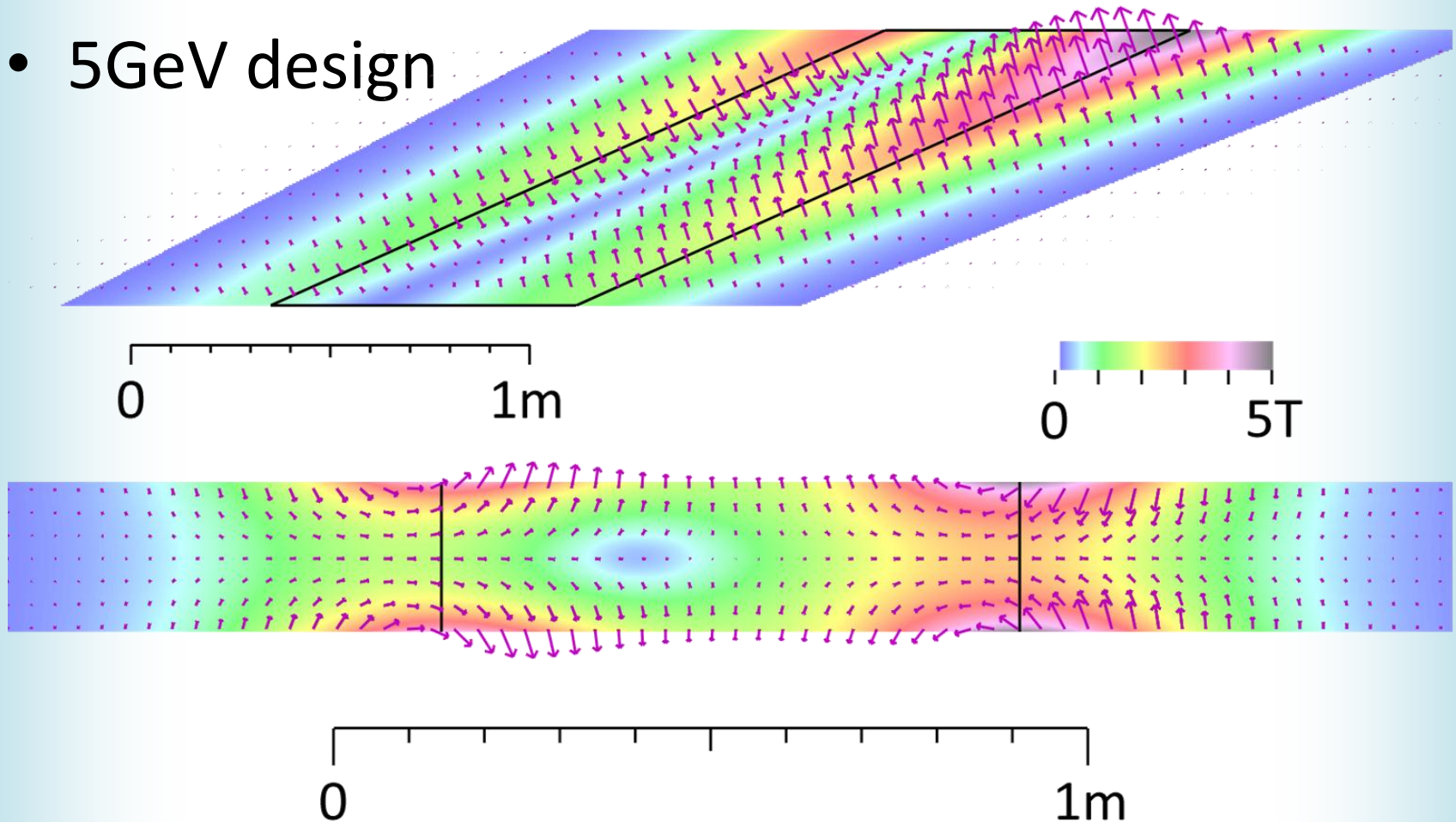
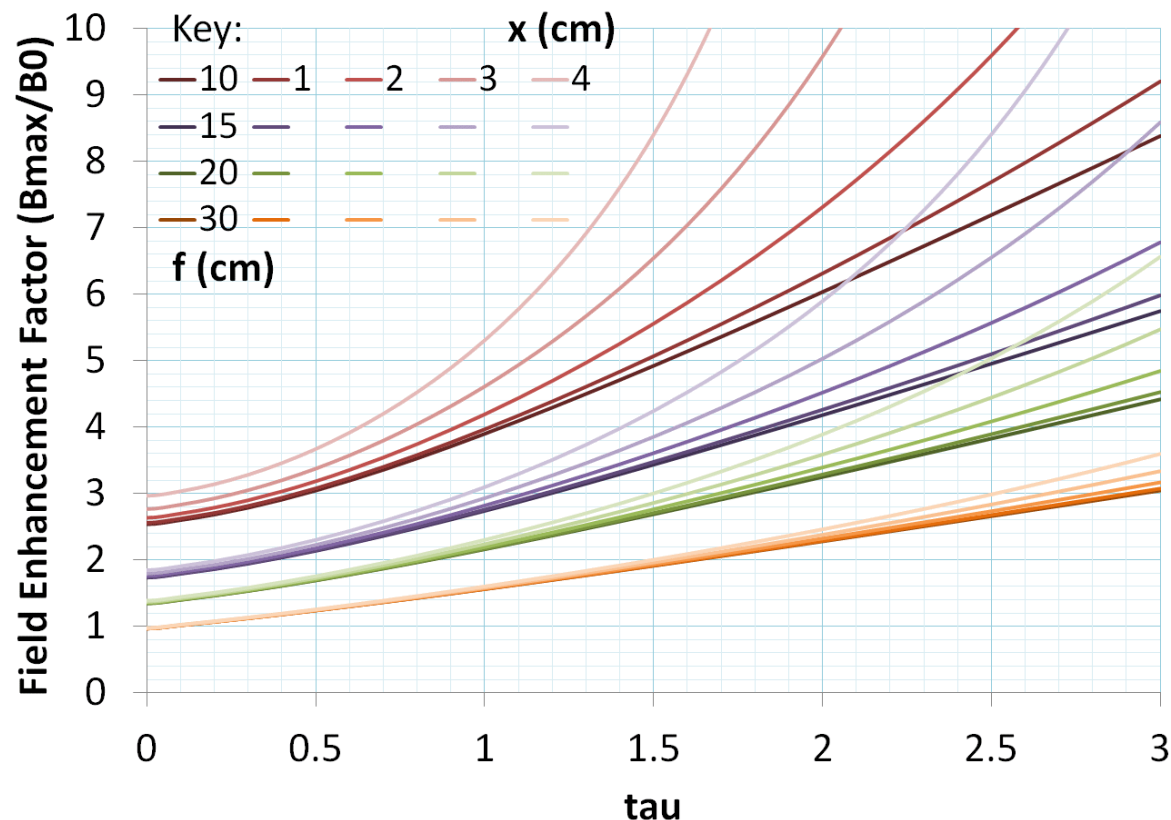


TABLE I. Transverse Parameters for VFFAG Rings

$E_{k,\text{inj}}$	800 MeV		
$E_{k,\text{ext}}$	3 GeV	5 GeV	12 GeV
Mean radius	52 m (2×ISIS)		
Superperiods	80 (superperiod is one cell)		
Cell length	4.0841 m		
Drift length	3.3174 m		3.1257 m
Magnet Parameters			
Magnet length	0.7667 m		0.9584 m
B_0	0.5 T		0.4 T
k	2.01 m^{-1}		2.2 m^{-1}
$\tau = \tan \theta_{\text{edge}}$	2.23		2.535
θ_{edge}	65.84°		68.47°
Fringe length	$f = 0.3 \text{ m}$ in $B \propto \frac{1}{2} + \frac{1}{2} \tanh(z/f)$		
B_{ext}	1.3069 T	2.0036 T	3.5274 T
$B_{\text{fringe}}/B_{\text{body}}$	$2.6941_{x=4 \text{ cm}}$		$2.6174_{x=2 \text{ cm}}$
B_{max}	3.5210 T	5.3979 T	9.2326 T
Beam Optics			
$y_{\text{ext}} - y_{\text{inj}}$	0.4780 m	0.6906 m	0.9895 m
μ_u (per cell)	71.30°		71.29°
μ_v	28.65°		19.56°
Q_u (ring)	15.843		15.843
Q_v	6.367		4.347

Field Enhancement Factor

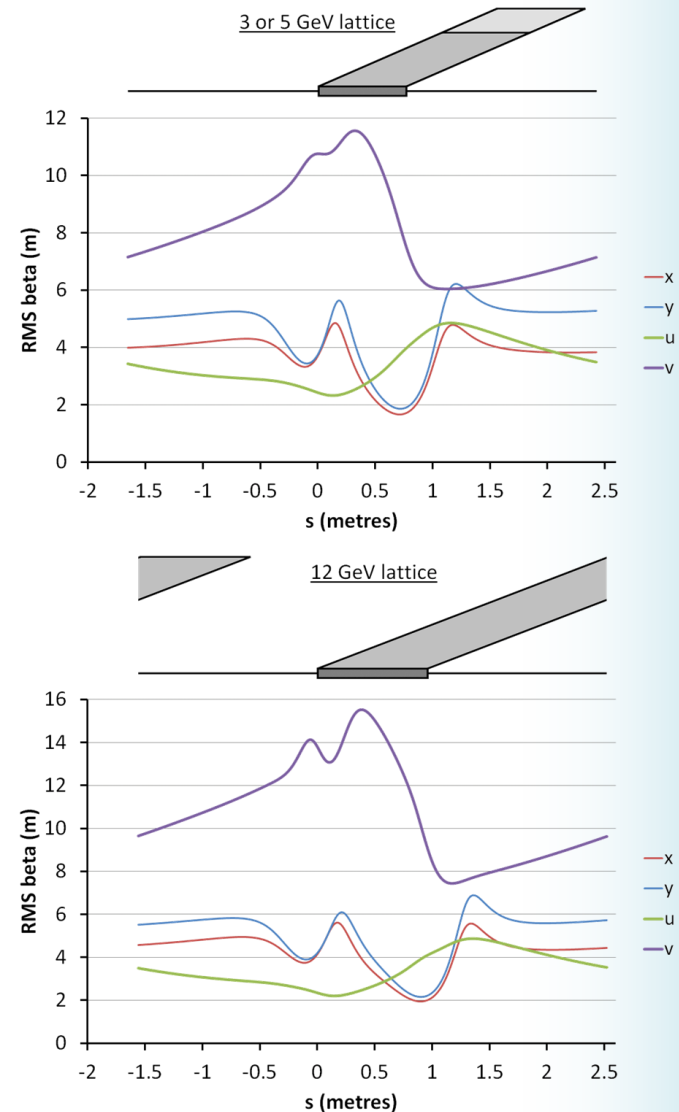
- For 3,5GeV designs with $k=2.01\text{m}^{-1}$

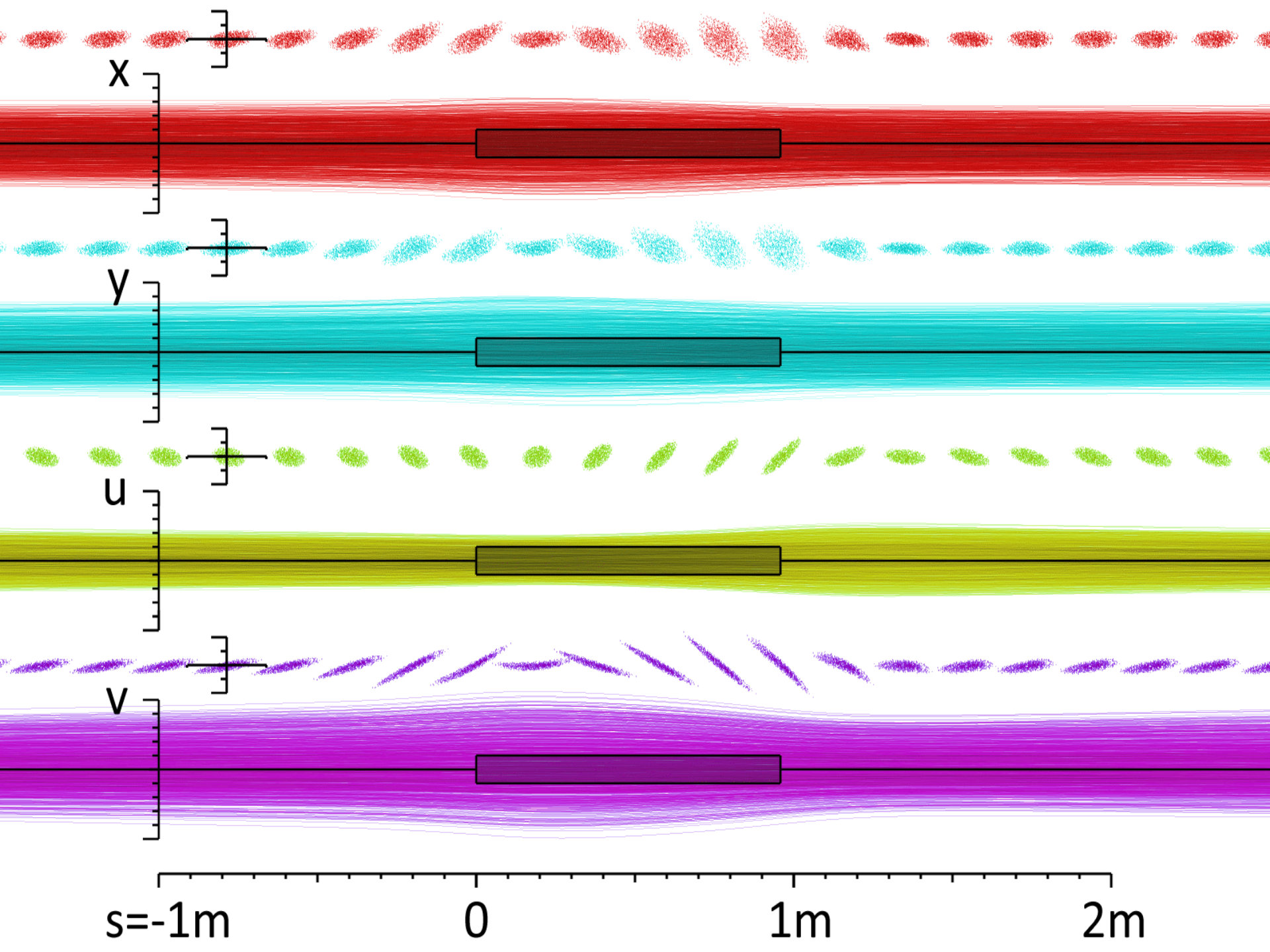


Cell Beta Functions

- Doublet focussing nature
 - Visible in u,v planes
- FfD
 - Doublet controlled by τ
 - Singlet controlled by k
- Ring tune sensitivity:

$$\frac{\partial Q_{u,v}}{\partial k} = \begin{bmatrix} -8.49 \\ -94.46 \end{bmatrix} \quad \text{and} \quad \frac{\partial Q_{u,v}}{\partial \tau} = \begin{bmatrix} 39.92 \\ 119.82 \end{bmatrix}$$





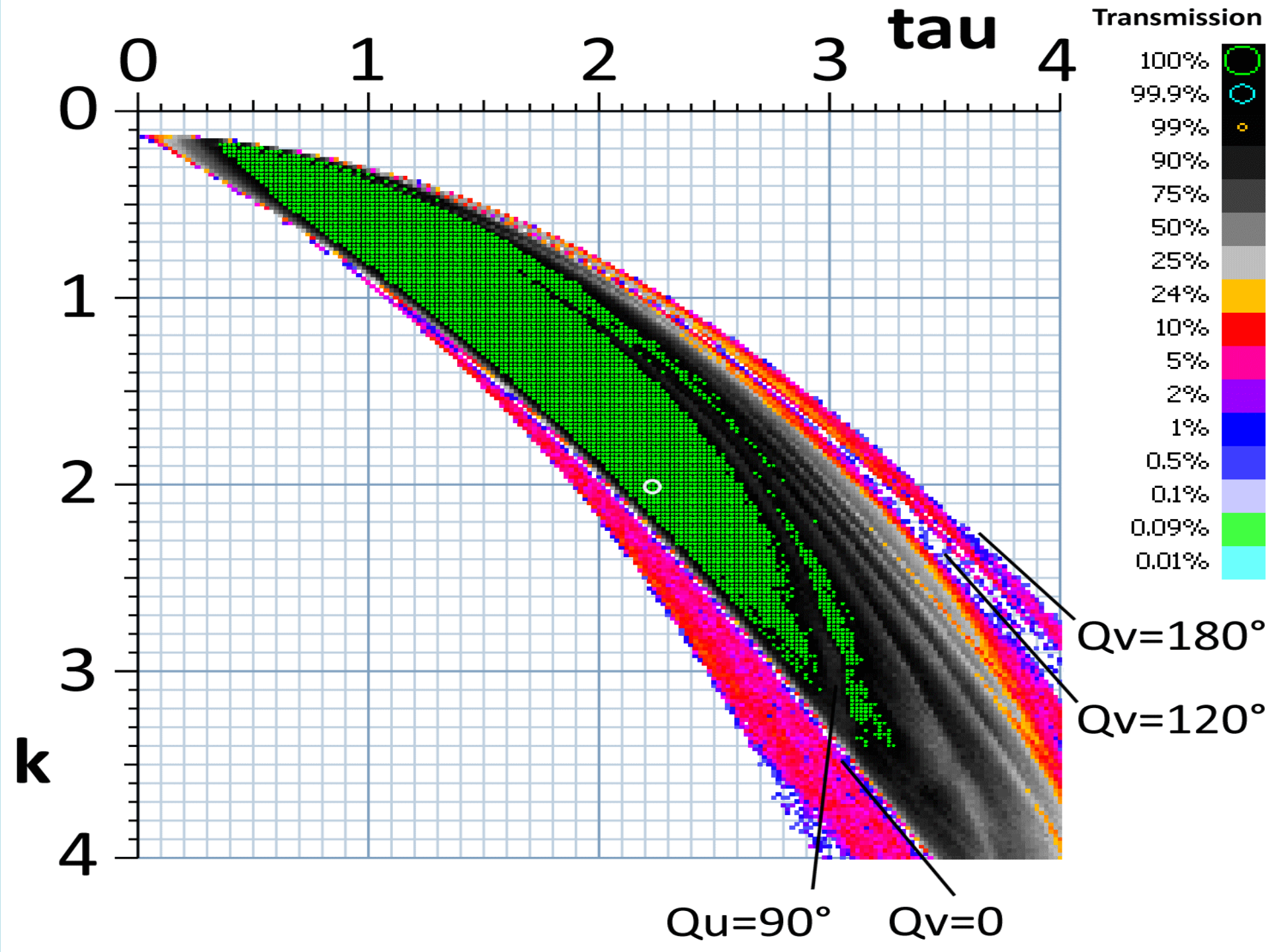
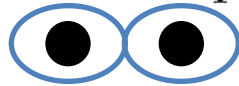

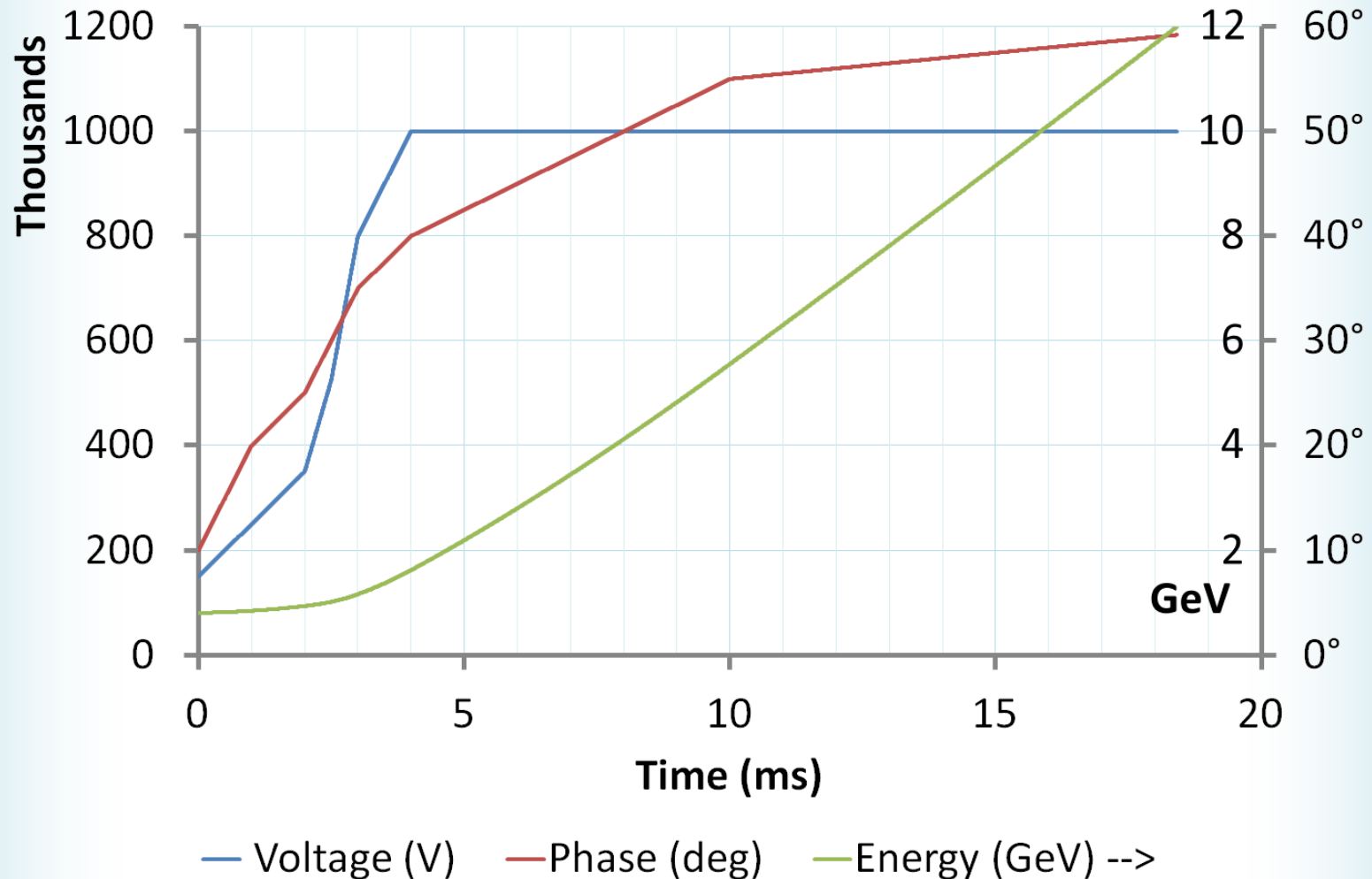


TABLE II. Longitudinal parameters for the 12 GeV VFFAG. Peak voltage per turn and phase are linearly interpolated from the times given.

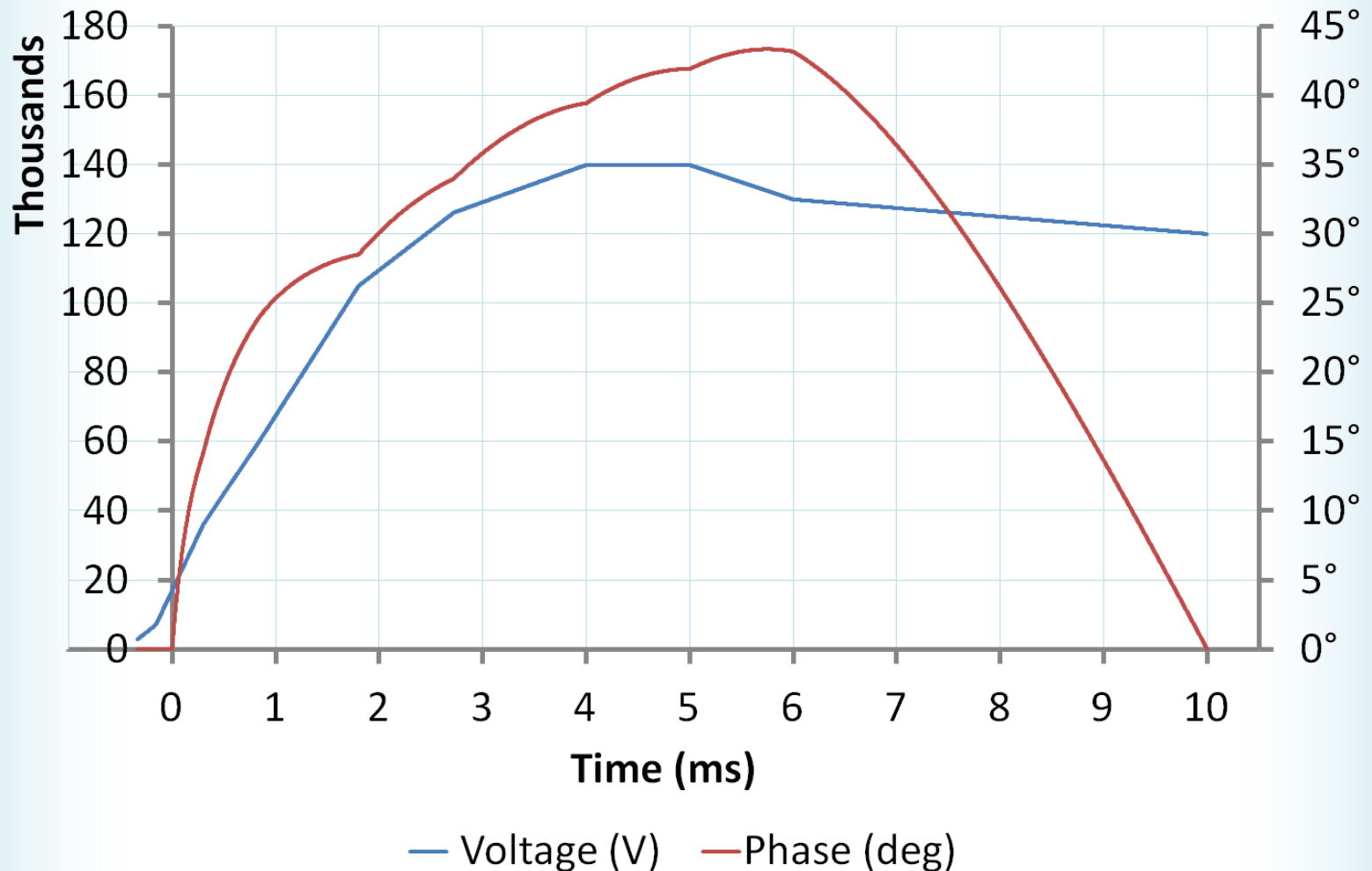


RF harmonic		$h = 8$
RF frequency		6.179–7.321 MHz
Cycle duration		18.41 ms
Rep. rate		50 Hz
Time (ms)	Voltage (kV)	Phase
0	150	10°
1	250	20°
2	350	25°
2.5	525	30°
3	800	35°
4	1000	40°
10	1000	55°
<i>18.41 (extract)</i>	<i>1000</i>	<i>59.21°</i>
20	1000	60°

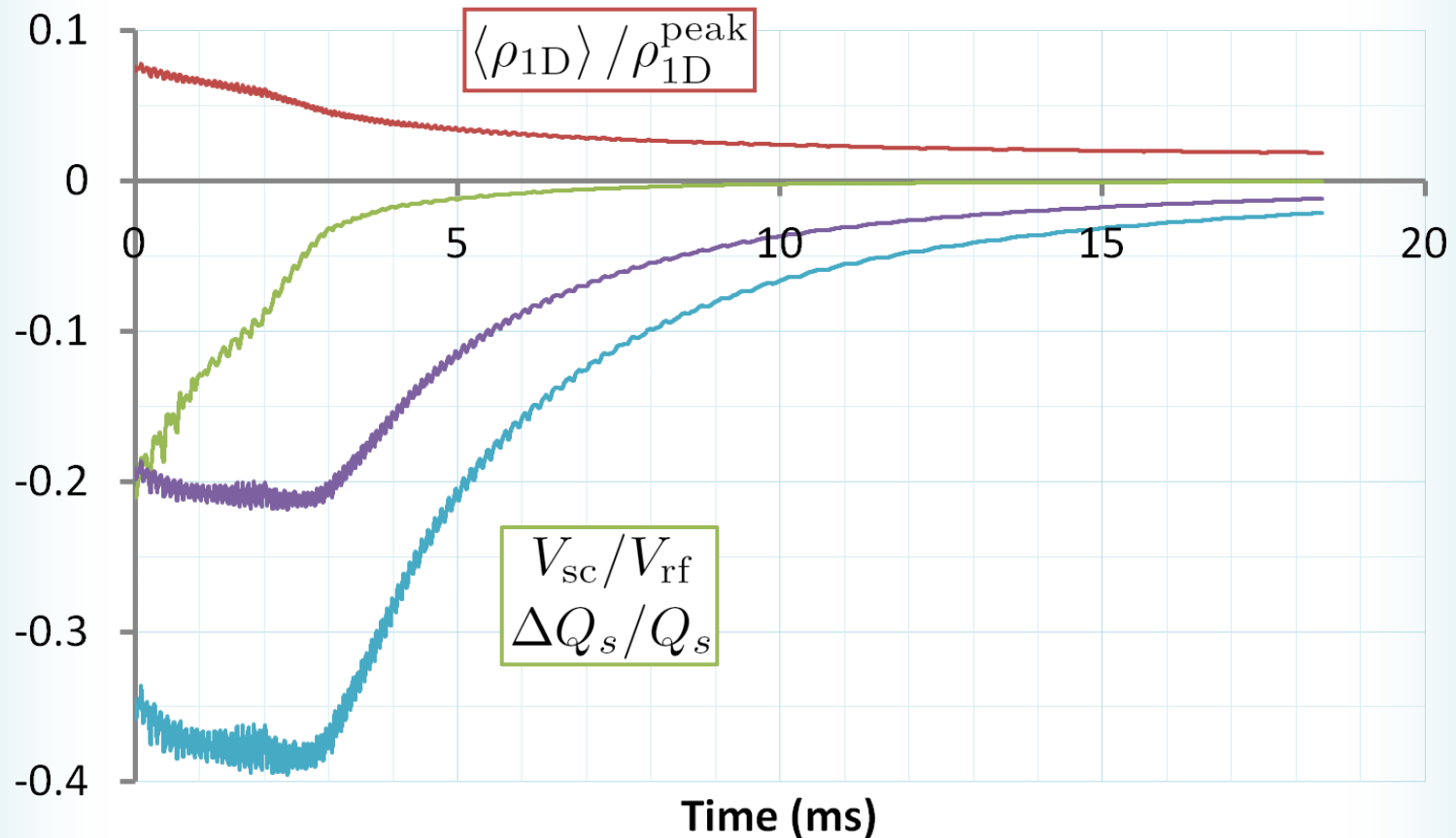
12GeV VFFAG RF Programme



Compare with ISIS 1st harmonic RF

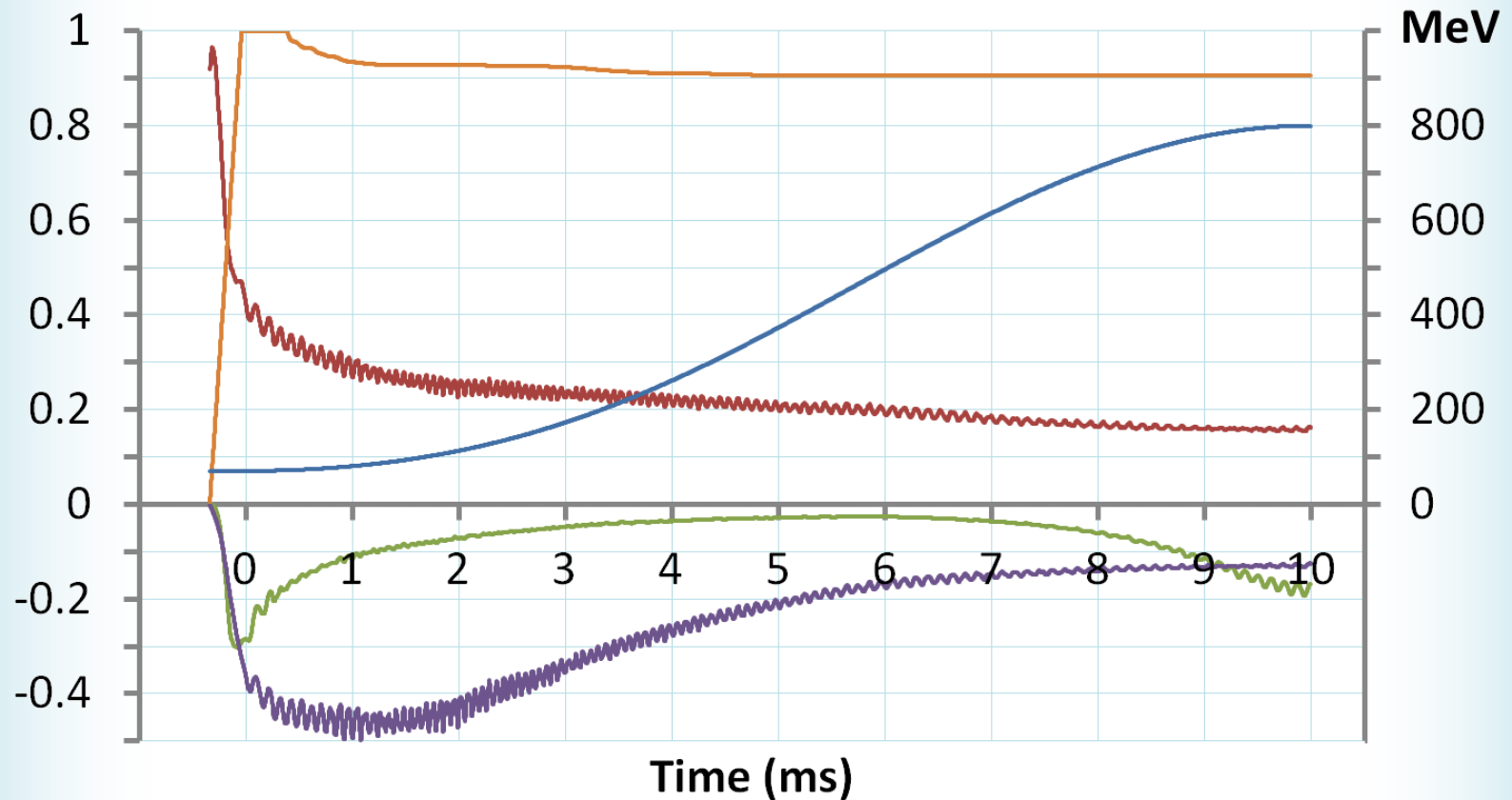


Longitudinal Intensity Effects



— Bunching factor — Space charge ratio — DeltaQu — DeltaQv

ISIS (1st harmonic) Intensity Effects

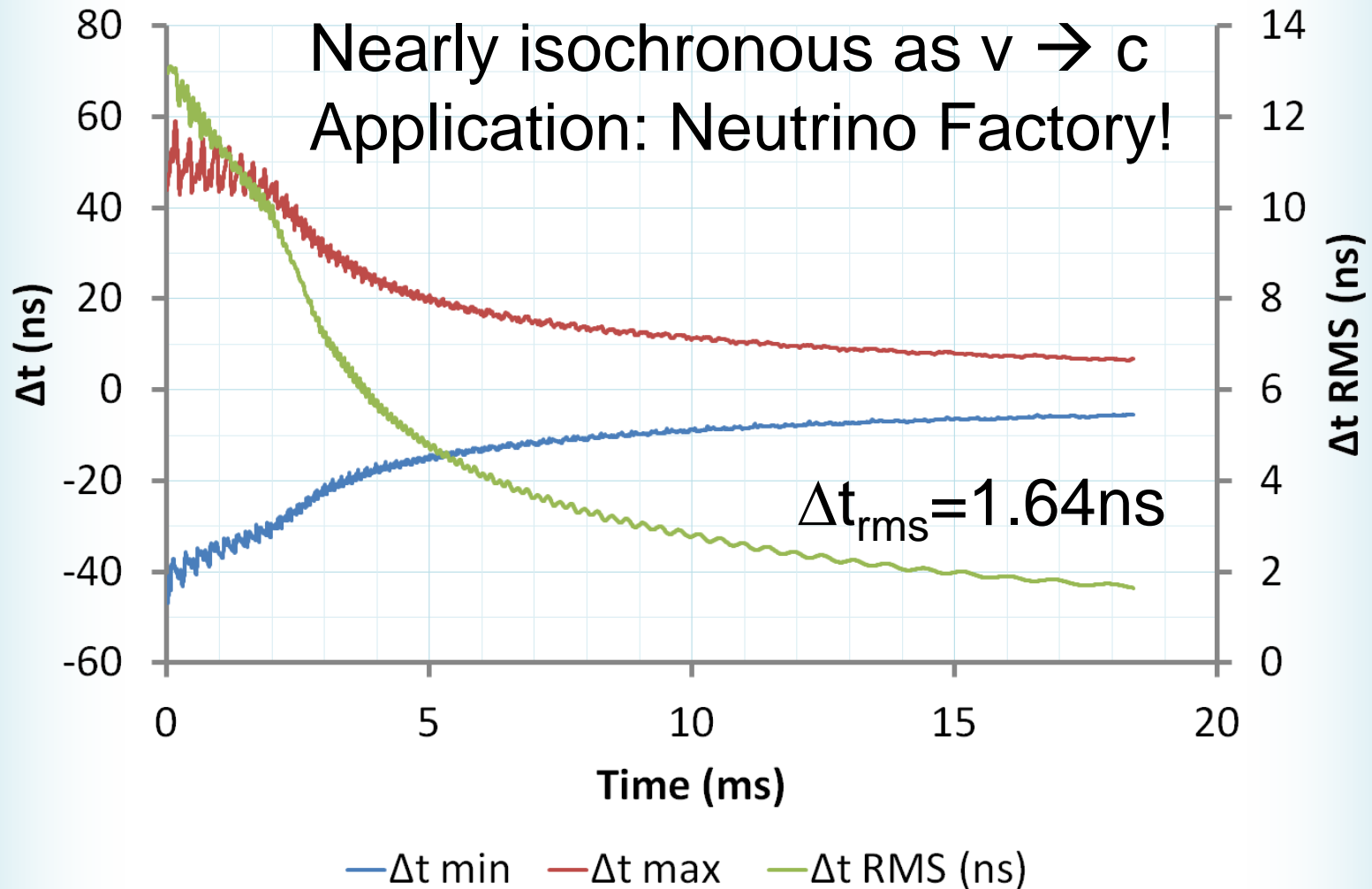


— Bunching factor — Space charge ratio — DeltaQx,y
— Transmission — Energy (MeV) -->

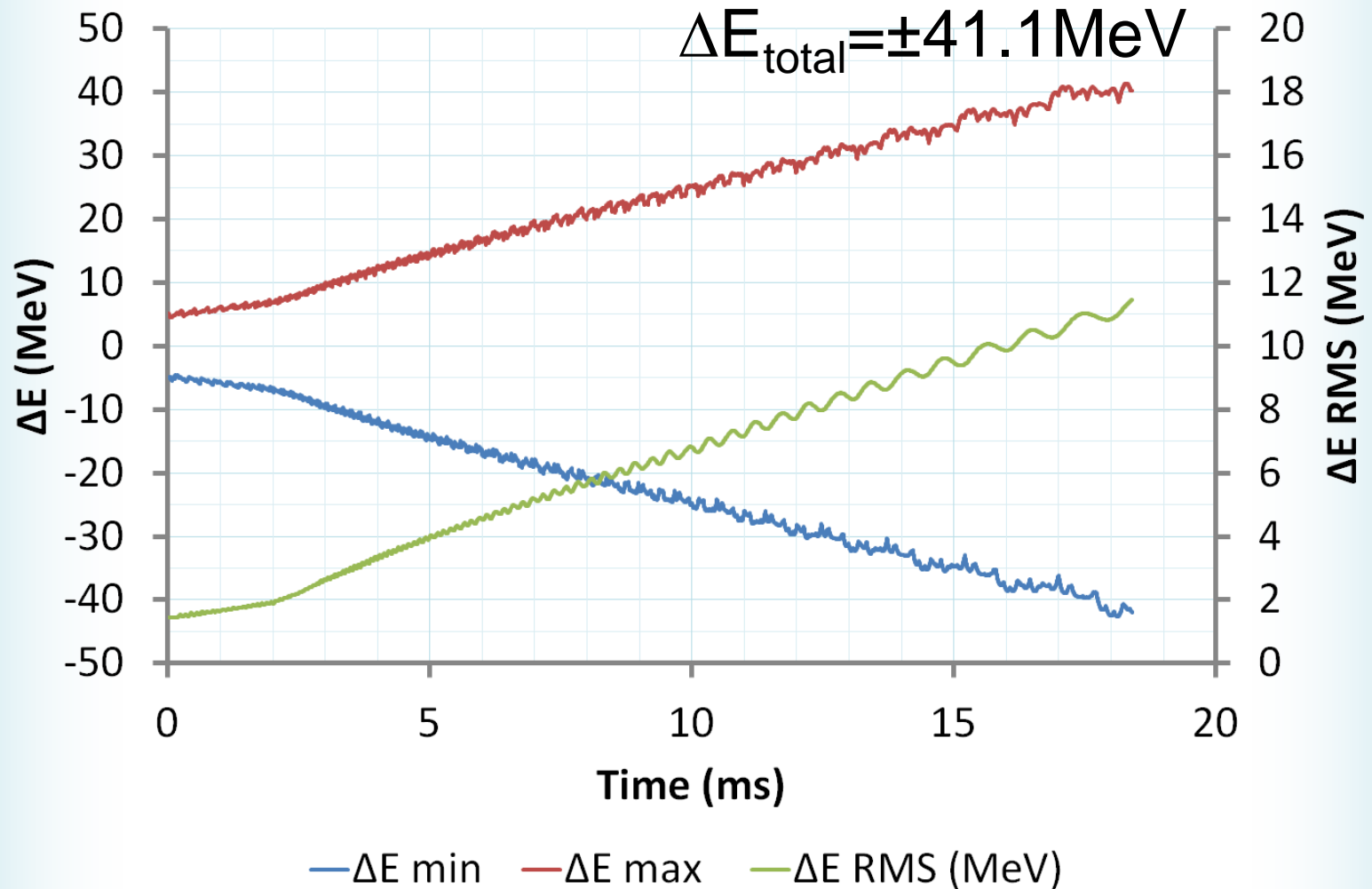
TABLE III. Intensity-dependent parameters for the ISIS single harmonic and 12 GeV VFFAG simulations run in series, for different numbers of protons injected into ISIS.

ISIS Protons In	2.50e13	2.75e13	3.00e13
ISIS μA in	200.3	220.3	240.3
ISIS transmission	90.54%	87.95%	85.98%
ISIS protons out	2.26e13	2.42e13	2.58e13
ISIS μA out	181.3	193.7	206.6
ISIS power (kW)	145	155	165
VFFAG transmission		100%	
VFFAG power (MW)	2.18	2.32	2.48
ISIS Peak Intensities			
Bunching factor	0.154	0.150	0.151
Space charge ratio	-0.301	-0.305	-0.311
$\Delta Q_{x,y}$	-0.499	-0.544	-0.580
VFFAG Peak Intensities			
Bunching factor	0.0188	0.0190	0.0190
Space charge ratio	-0.211	-0.257	-0.278
ΔQ_u	-0.219	-0.240	-0.254
ΔQ_v	-0.395	-0.434	-0.458

VFFAG Bunch Duration Evolution

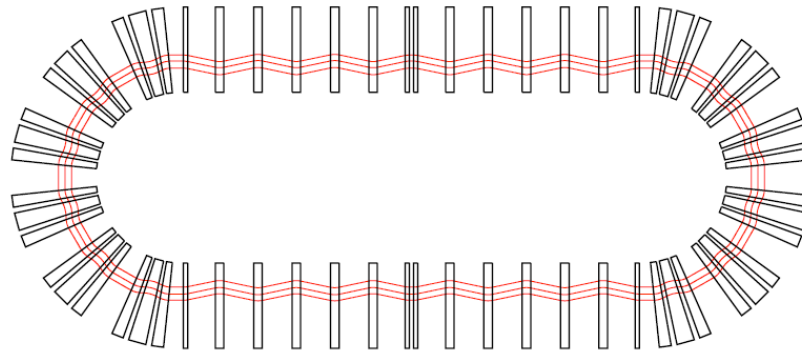


VFFAG Bunch Energy Spread



Proton Driver VFFAG Next Steps

- RF programme promising
 - But cavities won't fit in diagonal-shaped drifts!
- Adapt Yoshi Mori's idea of insertions in scaling FFAGs [Mori, FFAG11] to scaling VFFAGs



- Arc section using magnets with edge focussing
 - Straight section using FODO lattice, long drifts
- Then 2+1/2.5D simulation with space charge

III. Isochronous Machines

Tilted Orbit Excursion

- Any angle θ is allowed, not just vertical!
 - Quadrupole field will rotate by $\theta/2$
- Curved orbit excursion allows orbit radius \propto velocity

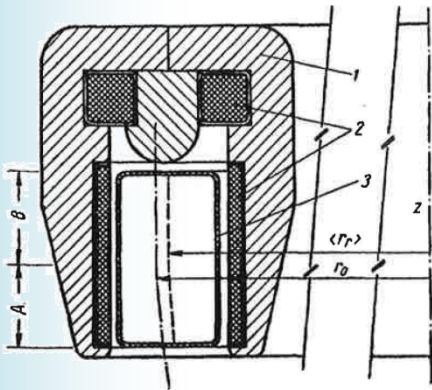
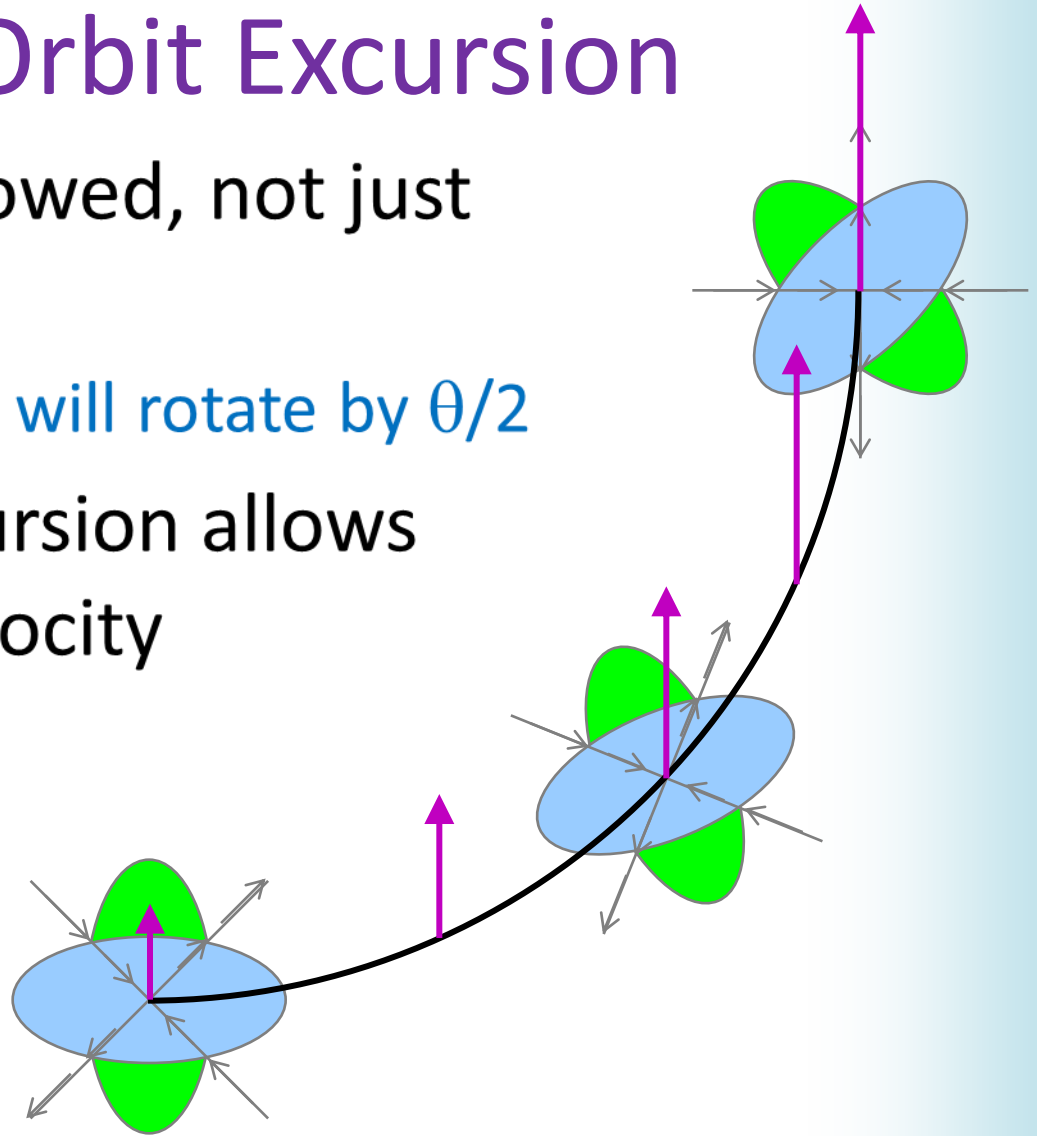


Fig. 1. Schematic section of accelerator with vertically increasing field; 1) ring magnet; 2) excitor windings for directing and focusing fields; 3) vacuum chamber; A) relativistic region; B) ultrarelativistic region.



← Teichmann (1962) also had idea

Analysis without Weak Focussing

- Mean radius $r = \beta R$ where $R = c/2\pi f$
- Mean $B_y = p/qr = m\beta\gamma c/q\beta R = \gamma(mc/qR) = \gamma B_0$
- For optics to scale, $B'l/p = \text{const.}$ ($B' = dB_y/ds$)
 - $B'l/p = B'r\Theta/m\beta\gamma c = B'\beta R\Theta/m\beta\gamma c = (B'/\gamma)(R\Theta/mc)$
 - $B' \propto \gamma \propto B_y$, therefore $B_y = B_0 e^{s/S}$ and $s = S \ln \gamma$
 - ...for some scaling length $S=1/k$
- To fix strong focussing tune, B_y must be exponential along the curved orbit excursion!

Lower Velocity Bound

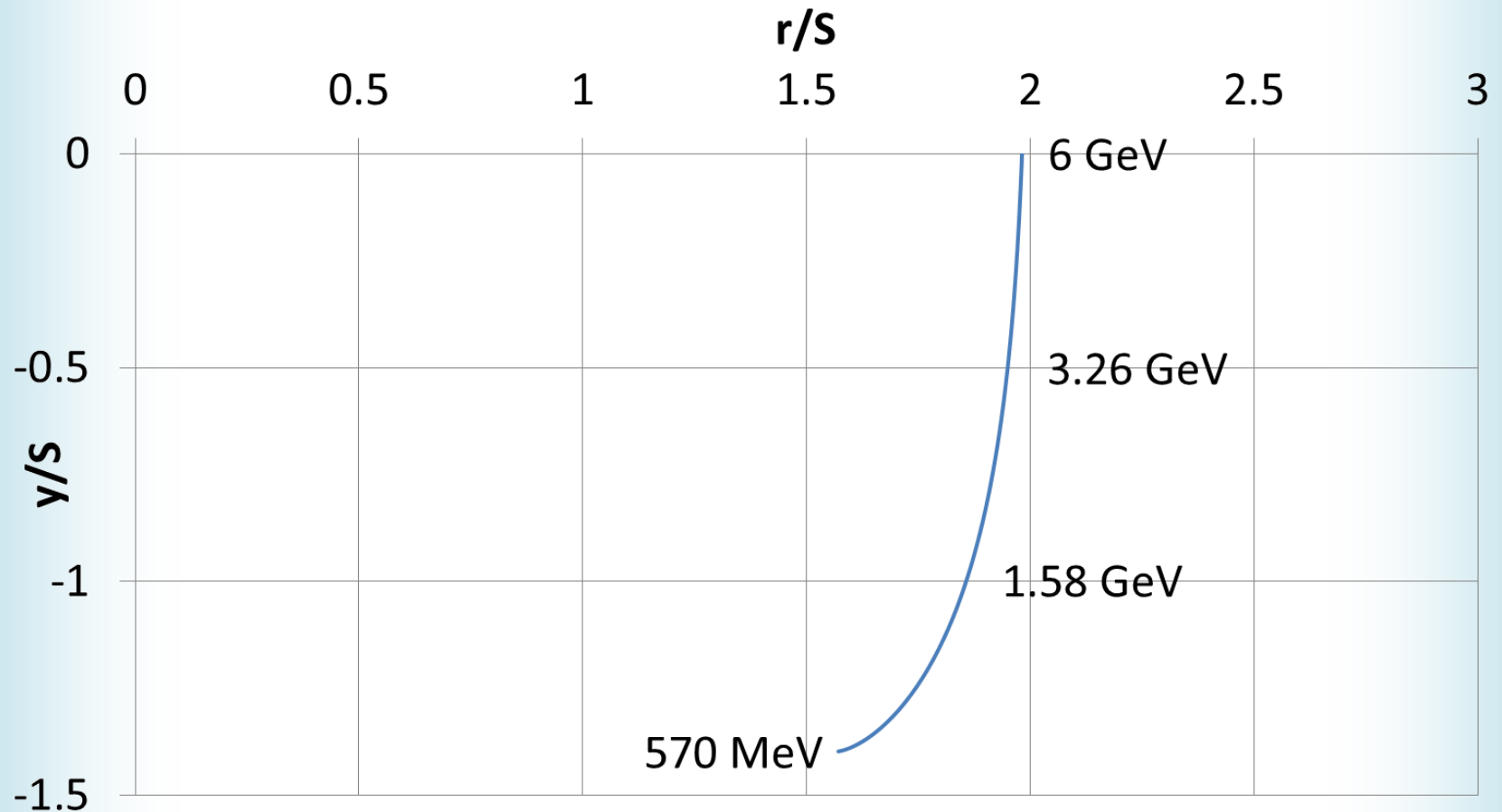
- Haven't yet used fact β , r are related to γ , s , B_y
- $r = \beta R = R\sqrt{1 - \gamma^{-2}} = R\sqrt{1 - e^{-2s/S}}$
- s is arc length so $dr/ds \leq 1$
 - Differentiate: $dr/ds = \frac{Re^{-2s/S}}{S\sqrt{1-e^{-2s/S}}} = \frac{R\gamma^{-2}}{S\beta} = (R/S)/\beta\gamma^2$
- Therefore $\beta\gamma^2 \geq R/S$ for scaling isoch. VFFAGs
 - Equality at horizontal excursion (minimum energy)
 - $dr/ds \rightarrow 0$ as $v \rightarrow c$ so asymptotically vertical
 - Compact machines require large R/S , high energy

Lower Energy Bounds for Protons

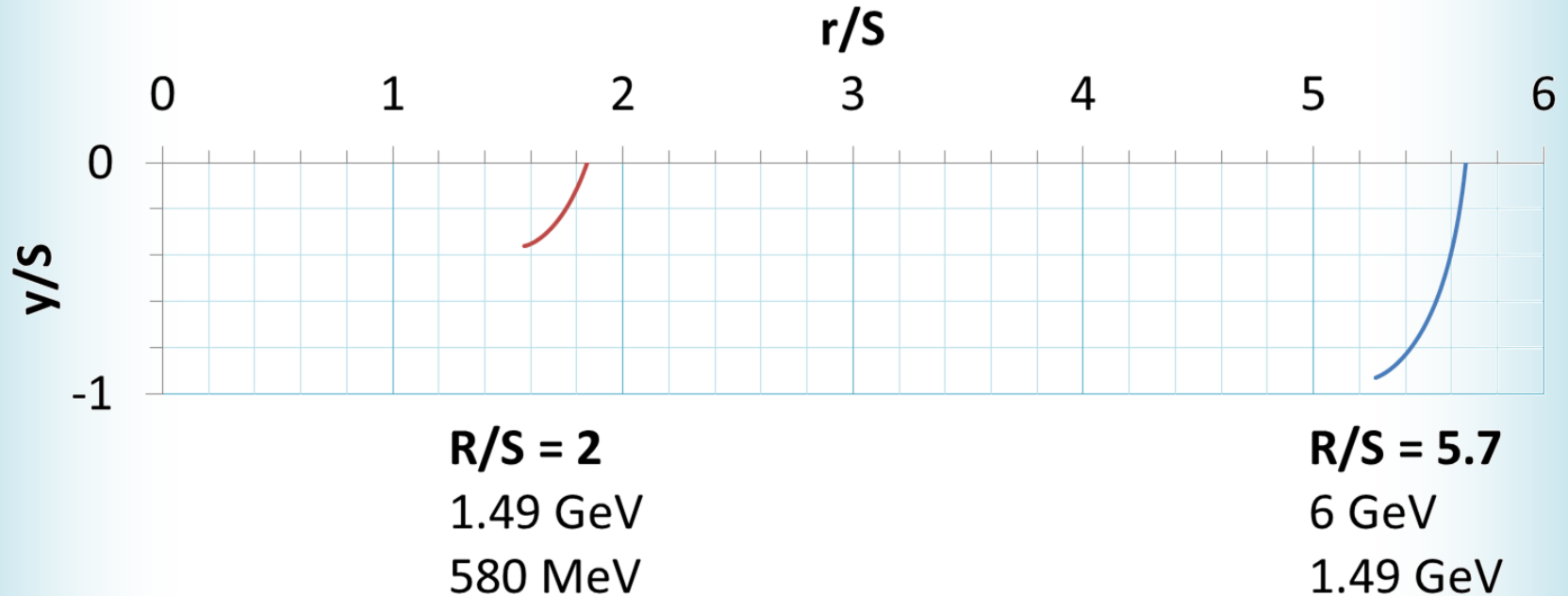
Minimum Proton Energy	β	Maximum R/S = $\beta\gamma^2$
100 MeV	0.428	0.52
200 MeV	0.566	0.83
500 MeV	0.758	1.78
1 GeV	0.875	3.73
2 GeV	0.948	9.29

- For muons or especially electrons, things are much easier!
- Can't join a cyclotron smoothly onto a *scaling* isochronous VFFAG with a different tune

Orbit Excursion Shape ($R/S=2$)



Staged Example



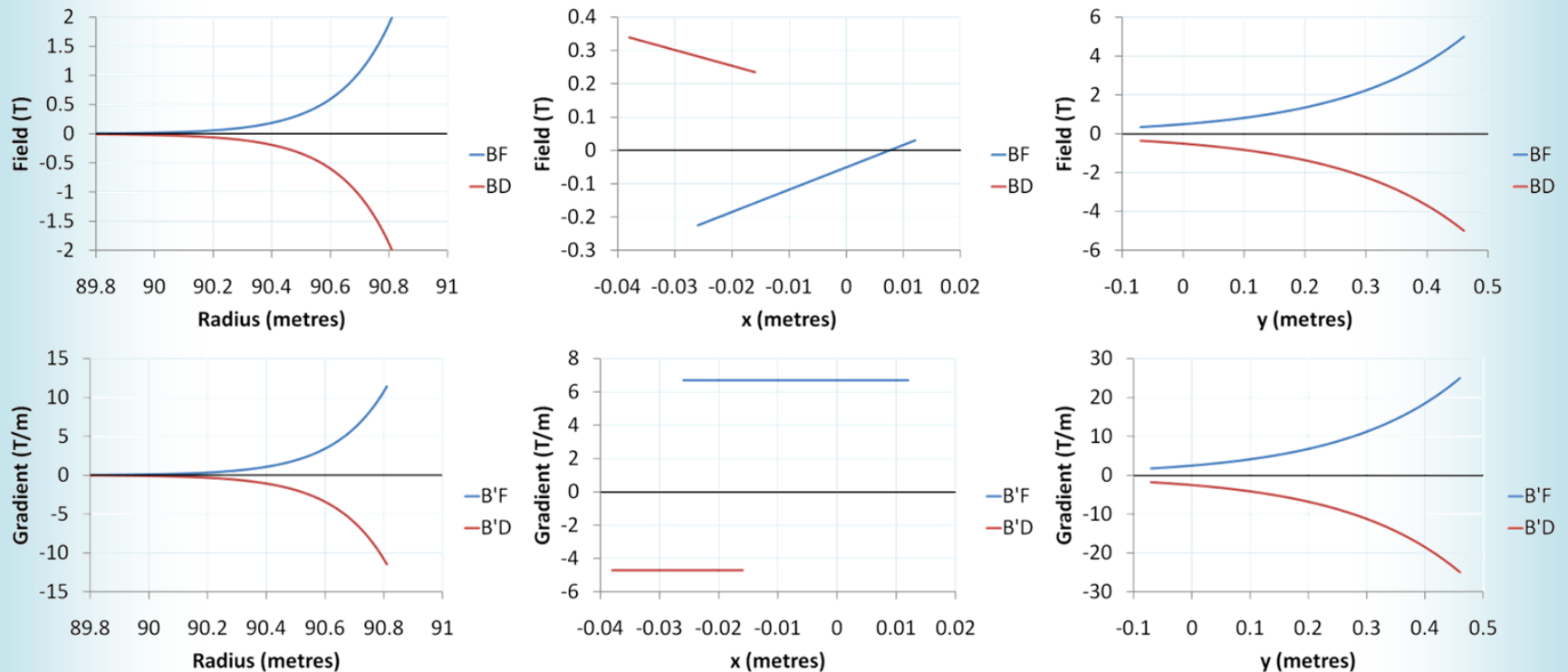
- If injector is PSI's 2.2mA 590MeV cyclotron, this two-VFFAG booster yields 13.2MW CW protons at 6GeV

Prospects for Isochronous VFFAGs

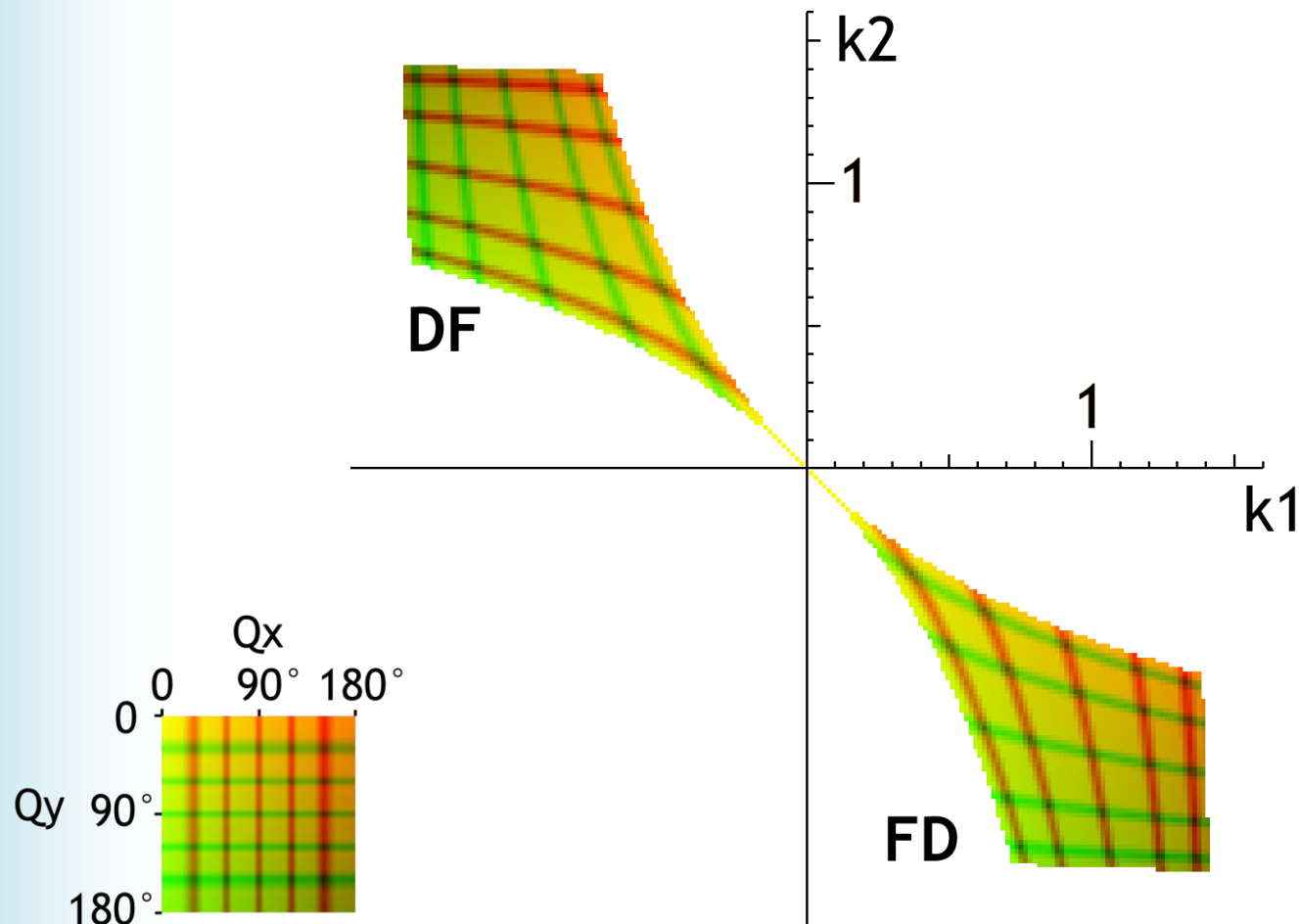
- I don't see these beating cyclotrons at ≤ 1 GeV
 - ...for protons.
 - Electrons: alternative to RCS?
 - Muons: alternative to non-scaling FFAGs?
- Protons at many GeV are potentially interesting for exotic particle factories
 - E.g. pbars, in terms of raw yield, though I believe most capture schemes assume a non-CW beam

IV. Three-Lens Horizontal FFAGs

Two Magnet Families Only

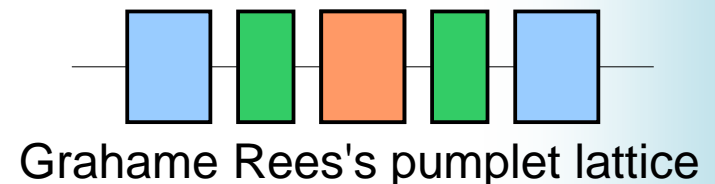
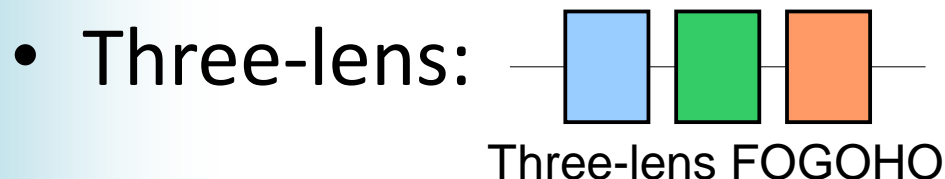
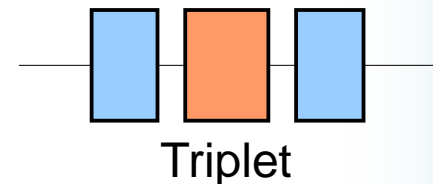
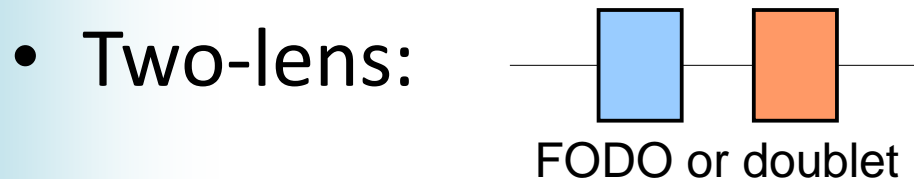


Two-Lens Stability Diagram

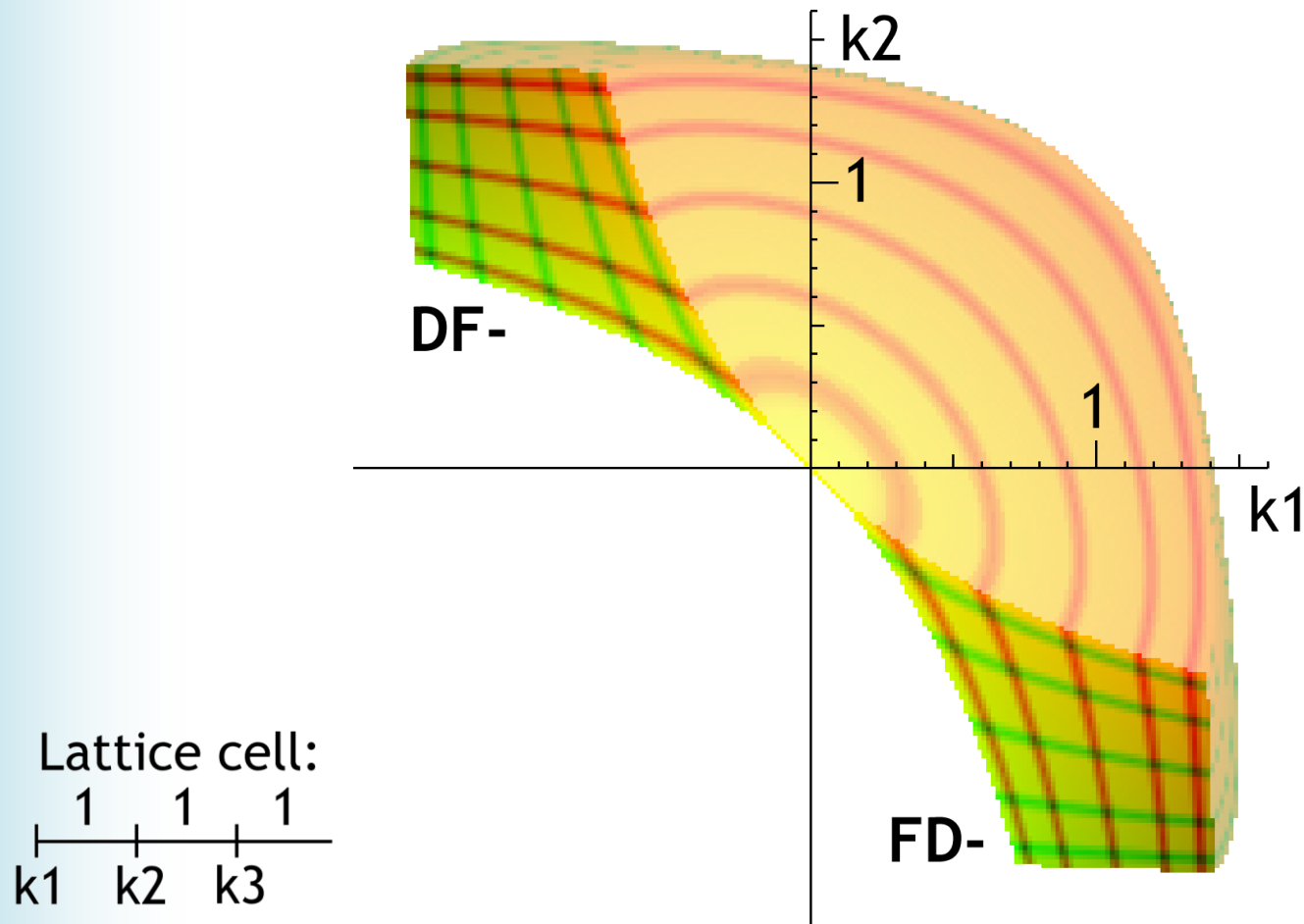


Three-Lens Lattice Advantages

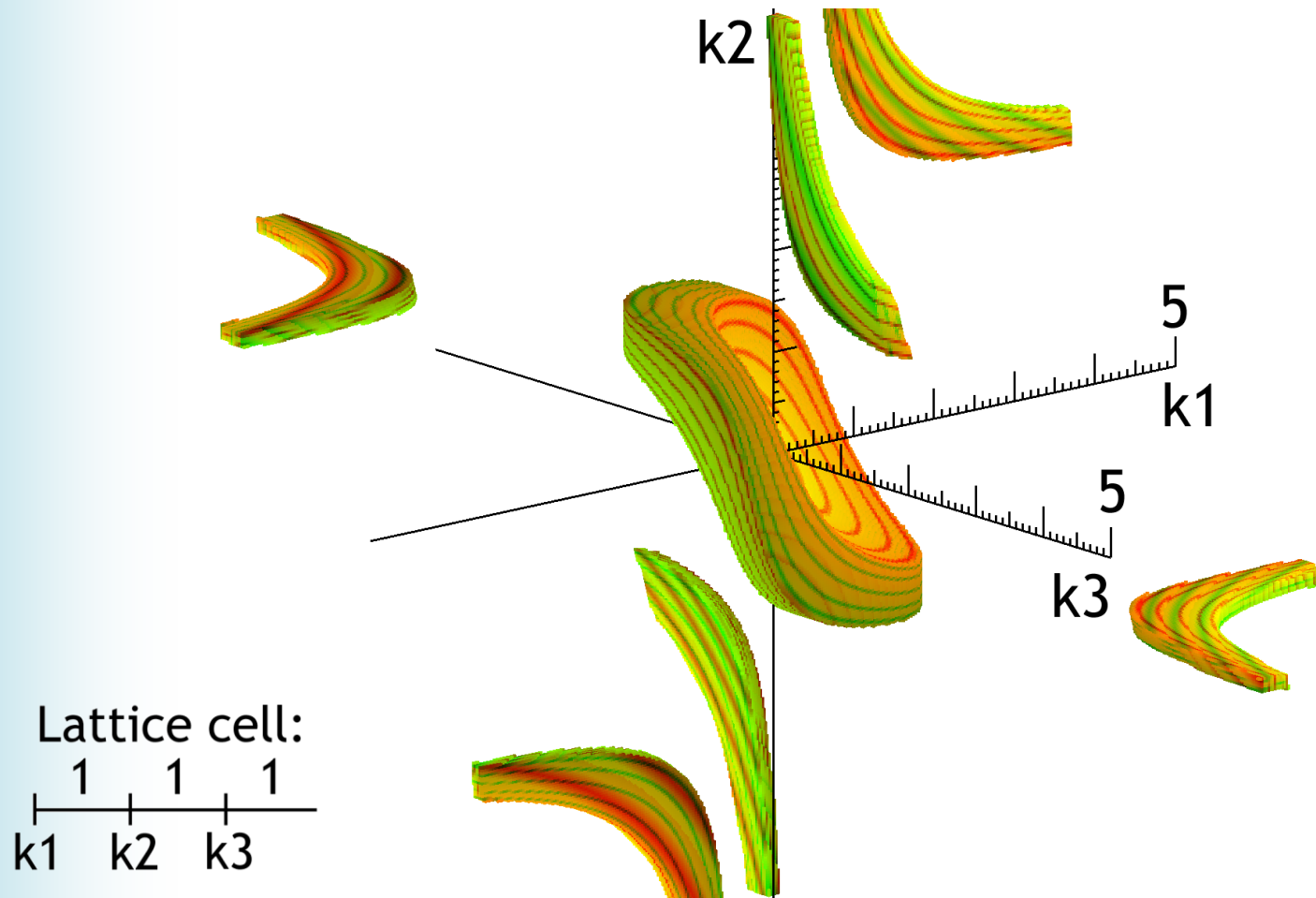
- Extended energy range non-scaling FFAGs
 - Allows gradient reversals e.g. FFD changing to FDD
- Fixed-tune non-scaling FFAGs
 - With at least three free gradients, you can satisfy
 - $dQ_x/dp = dQ_y/dp = 0$
 - ...and not become a scaling FFAG!



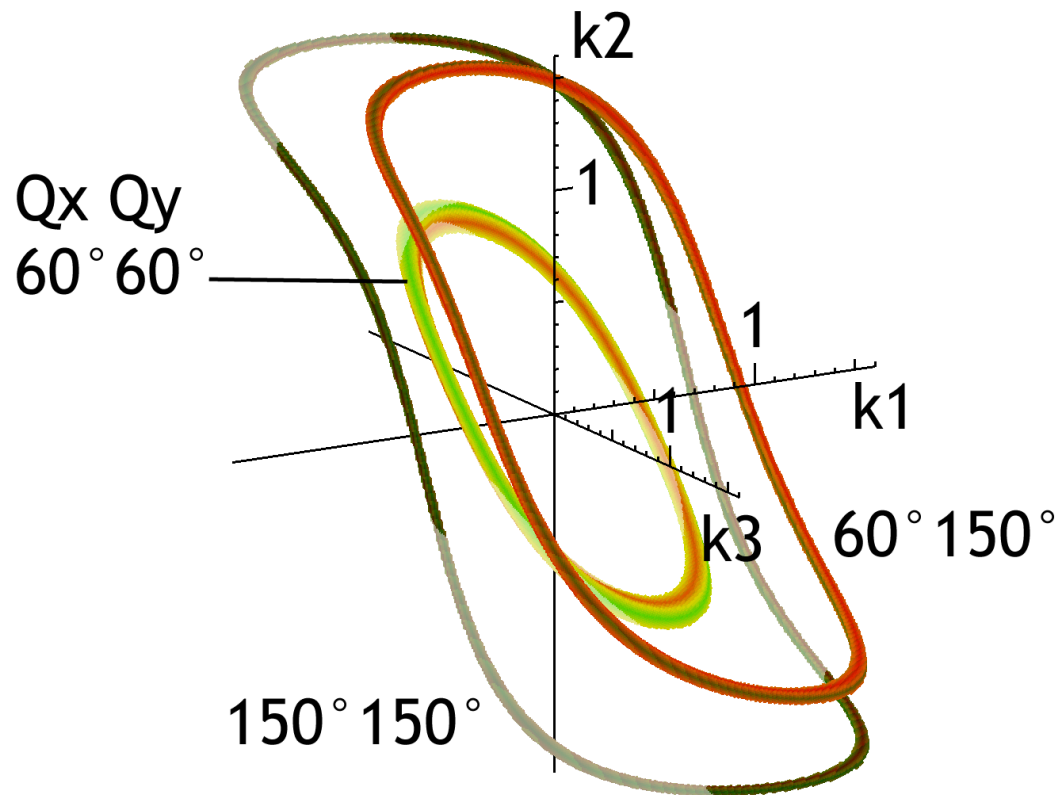
3D “Necktie” Diagram



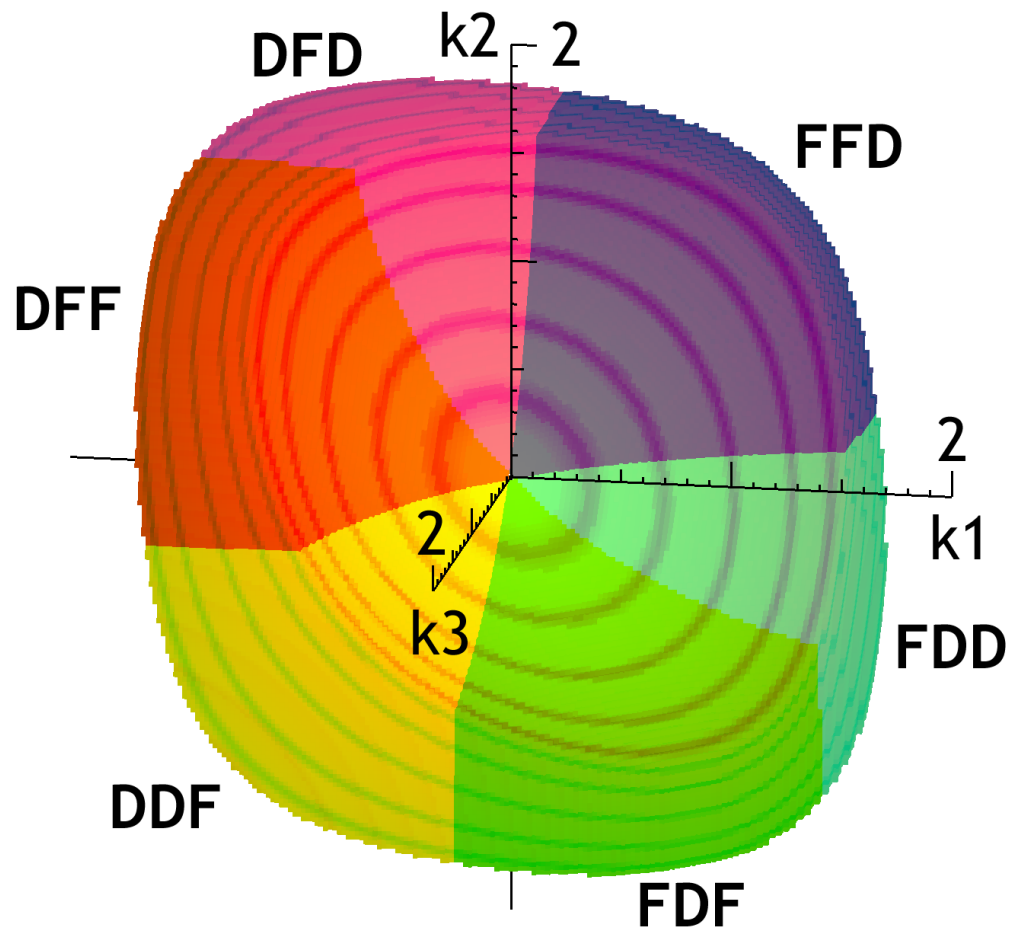
Three-Lens Stability Regions



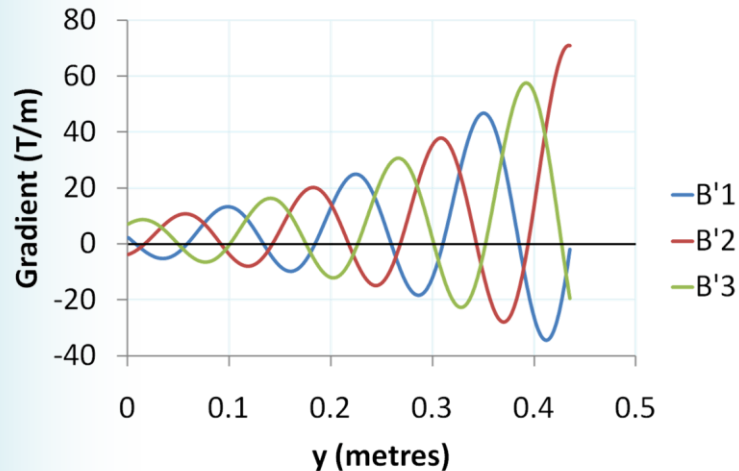
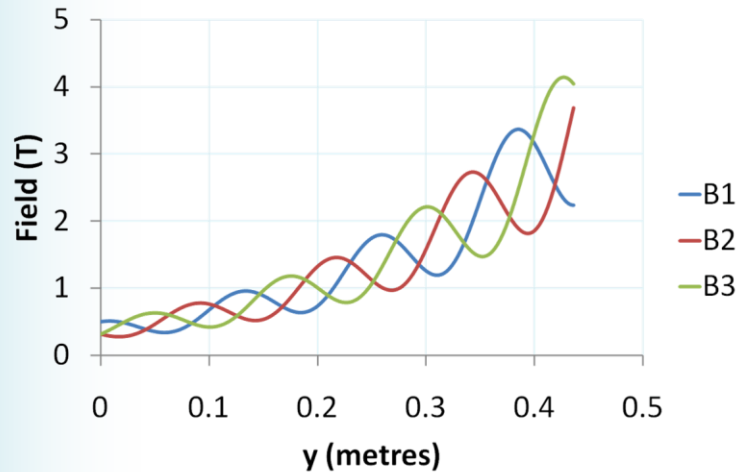
Loci of Fixed Tune



Change in Nature of Lattice



[IPAC11] attempt at 3-lens VFFAG



- Maybe somehow all magnets can bend the right way?

$$B_{y,n}(0, y) = B_0 e^{ky} (1 + a \cos(wy + \varphi_n))$$

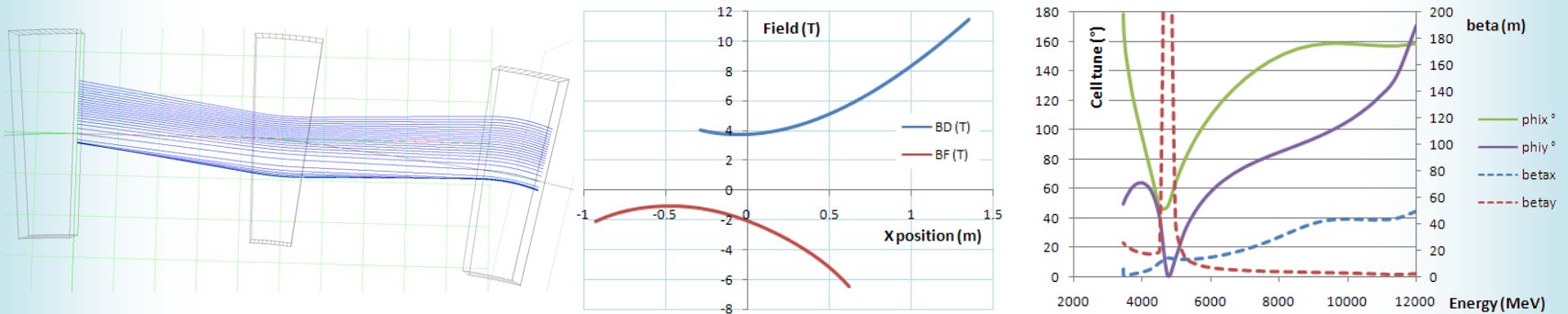
- Didn't work

[PAC09] Optimiser designed FFAG

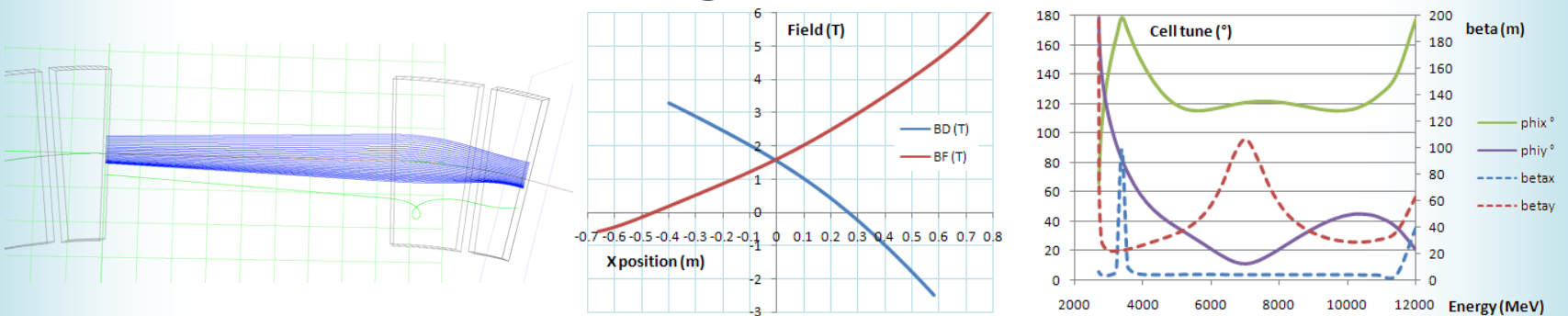
- Before the VFFAG idea, I tried to make a non-scaling (horizontal) FFAG 12GeV proton driver
- Used the Muon1 optimiser (evolutionary algorithm) on field polynomials $B_{y,n}(x)$
 - Previously used on neutrino factory
 - <http://stephenbrooks.org/muon1/> distributed project
- Scored 0-100 for percentage of energy range with stable closed orbits
- Didn't work

FFAGs not working in more detail

- FODO cell not working:



- Doublet not working:



(2013) Combine the two ideas

- Use the optimiser on a 3-lens horizontal FFAG
 - Just add a third magnet to the proton driver cell
- Score 0-100 for energy range with stable orbit
 - and because we're being optimistic,
- Score 100-200 for flat cell tunes
 - $\text{score} = 200 - 100 (Q_{x,\text{max}} - Q_{x,\text{min}} + Q_{y,\text{max}} - Q_{y,\text{min}})$
 - Provided the full energy range is stable
- Final score: 196.601

Muon Transmission (%)

180

160

140

120

100

80

60

40

20

0

1000

2000

3000

4000

5000

6000

7000

8000

9000

10000

11000

12000

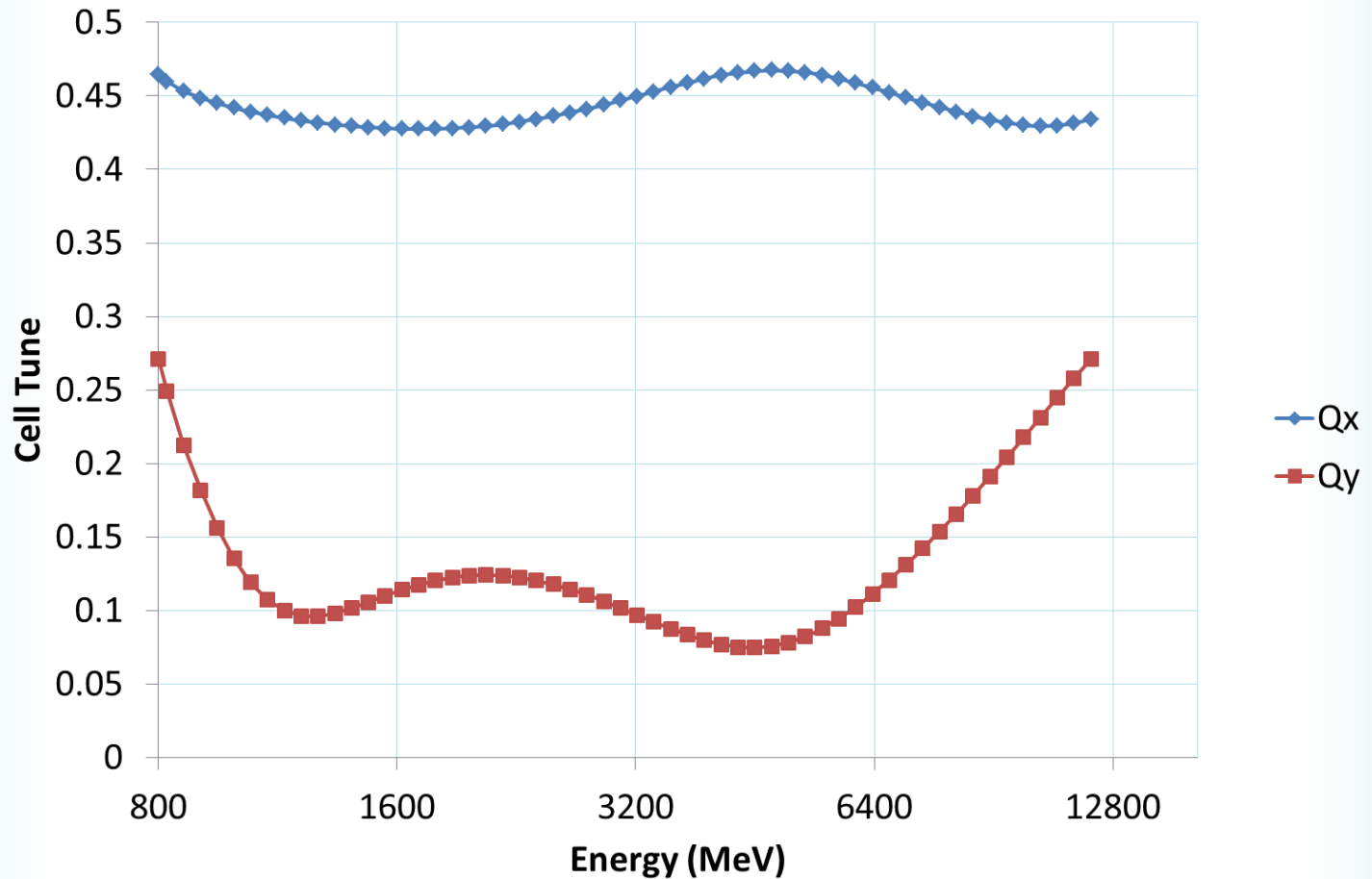
13000

14000

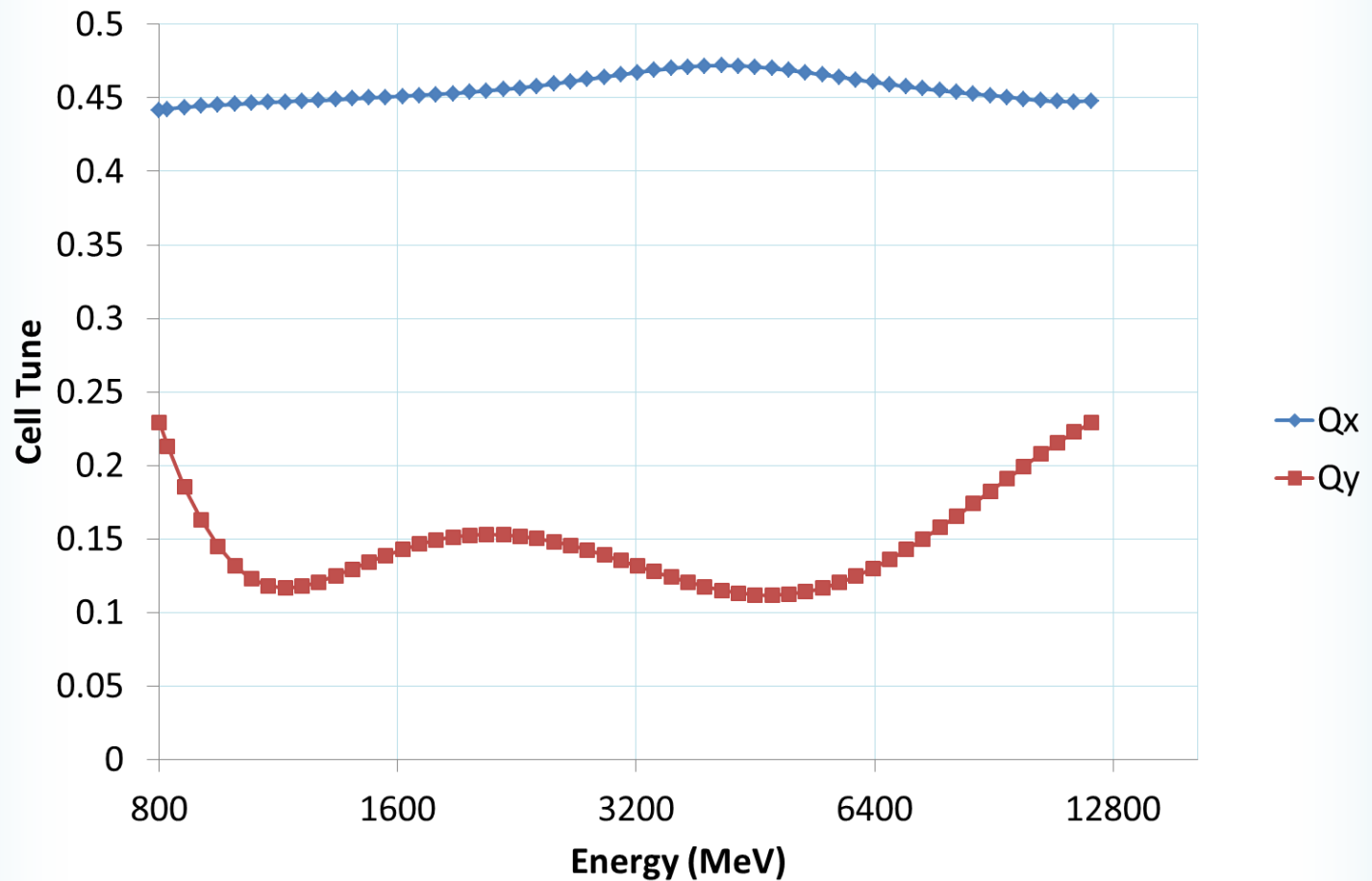
Results

Random
Mutate
Crossover
Interpolate
Extrapolate
MuSpherical
MuOn
Extremes
LocalGrasp
TopoSmooth
CellDouble
CellDelete
CellCommute

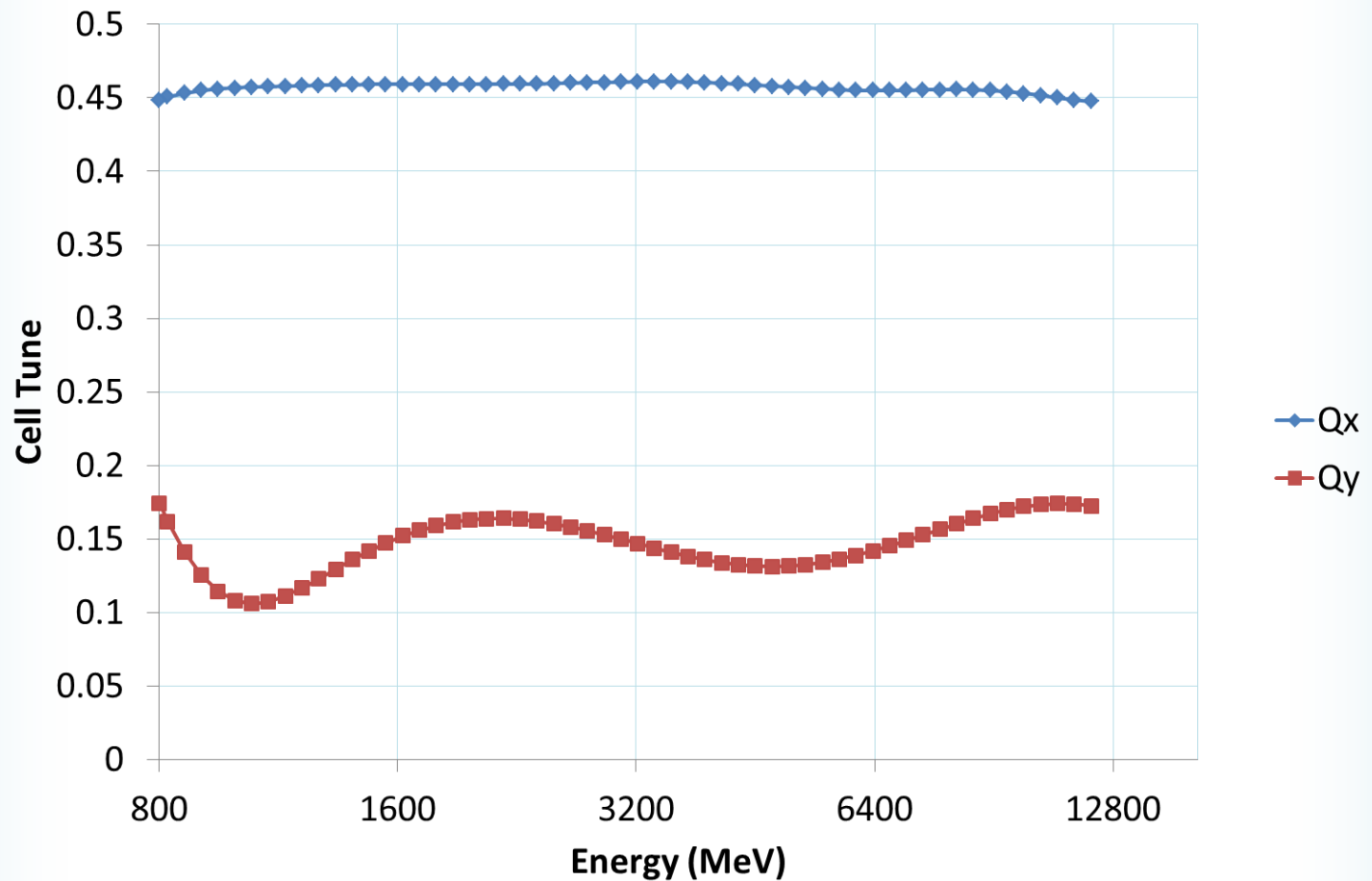
Tune Variation, score = 176.379



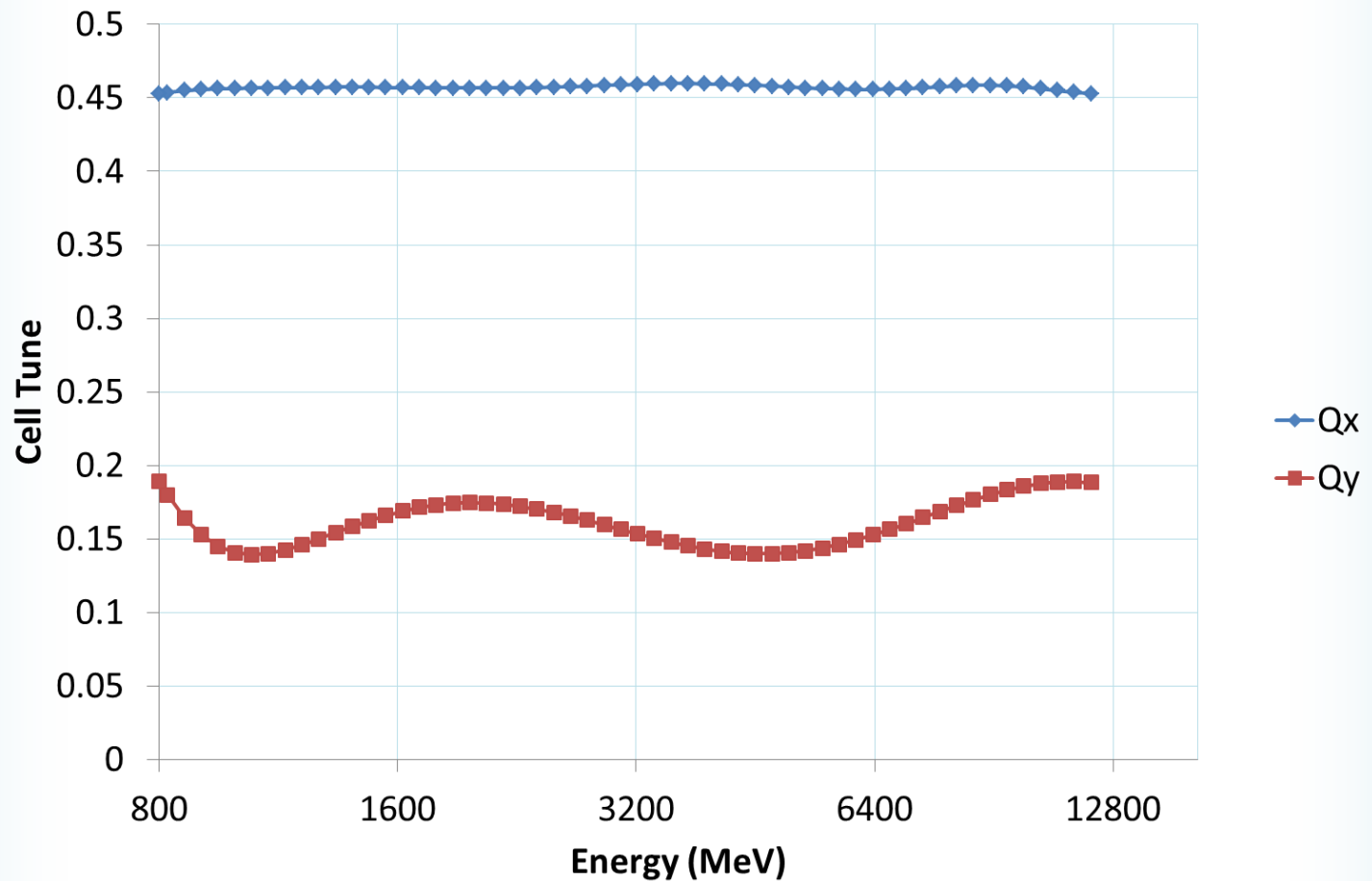
Tune Variation, score = 185.216



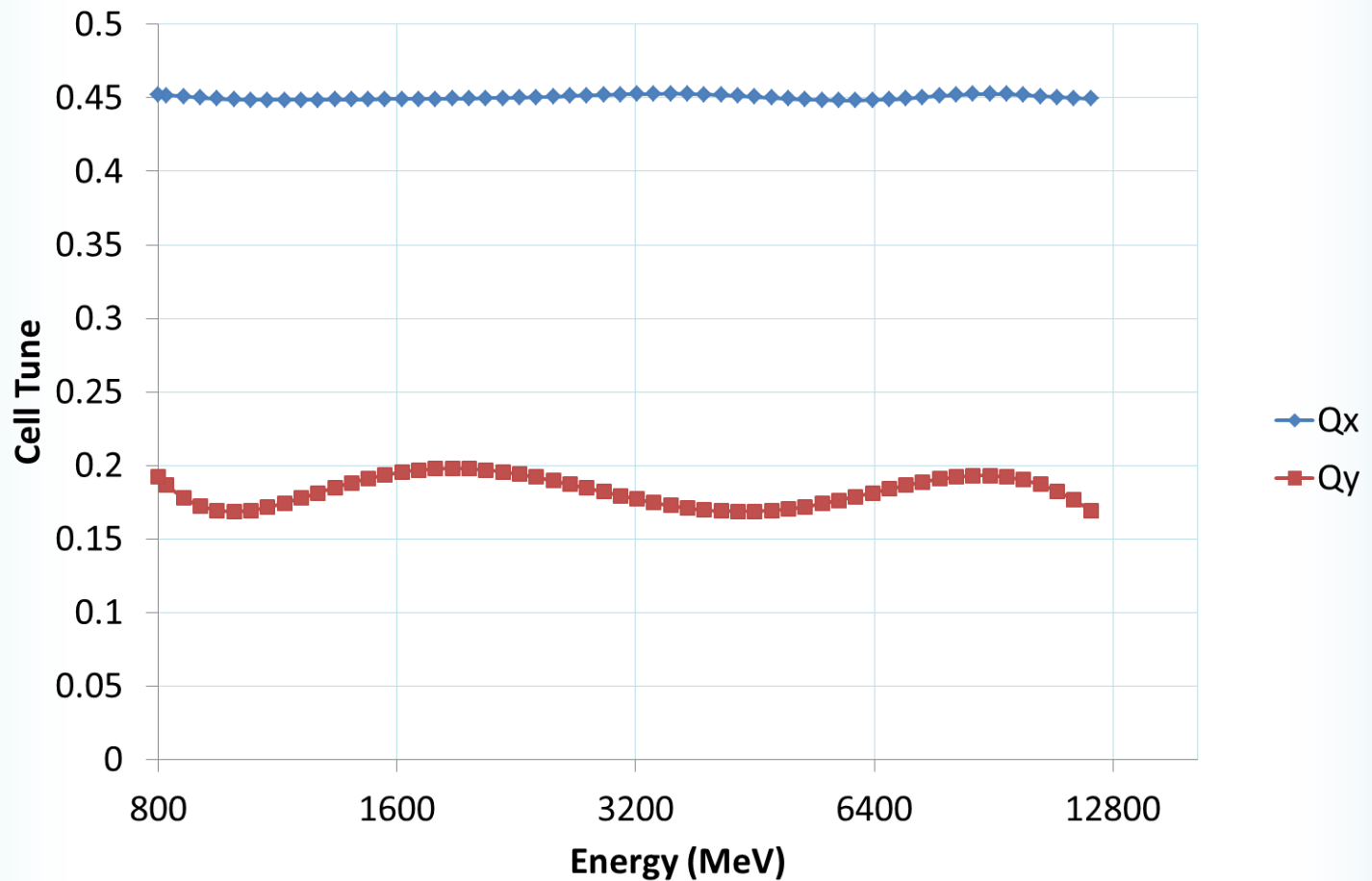
Tune Variation, score = 191.844



Tune Variation, score = 194.263

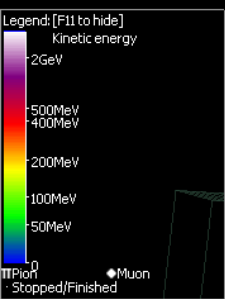


Tune Variation, score = 196.601




```
t = 99999999999999999900000000.000 ms magnet3test4fogoho  
Beam parameters: na% Otherwise lost: na% Wrong way: na% save in 3m09s
```

Frame-rate: 1/1 Autoview ON Particle size: AUTO (5mm)
Results database: 39804 entries, 18.00 MB (18.00 MB since last send)



$E = 800 \text{ MeV}$

0.46469	4.39746
-0.47833	-2.37458
0	0
0	0

4.39746
-2.37458
0
0

```
1.68648e-006
-9.70404e-007
-0.95416
-0.39887
```

1.72262e-006
2.05749e-006
6.48779
1.66406

```
[eps=1e-006]
Coupling = 6.437e-006
det(x) = 0.99999 det(y) = 1
Qx = 0.45204 betax=14.817 m alphax=4.7834
Qy = 0.19225 betay=6.93966 m alpay=-1.40029
```

Drift D3 BH
Length=2.15548

Drift DI
Length=3.40665

BF

Drift D2
Length=197089



Drift DS
Length=2,15548

Drift D1
Length=3.40665

```
xi[0] = -0.38443
xi[1] = -0.12095
xi[2] = 0
xi[3] = 0
```

```

xo[0] = -0.38443
xo[1] = -0.12095
xo[2] = 0
xo[3] = 0

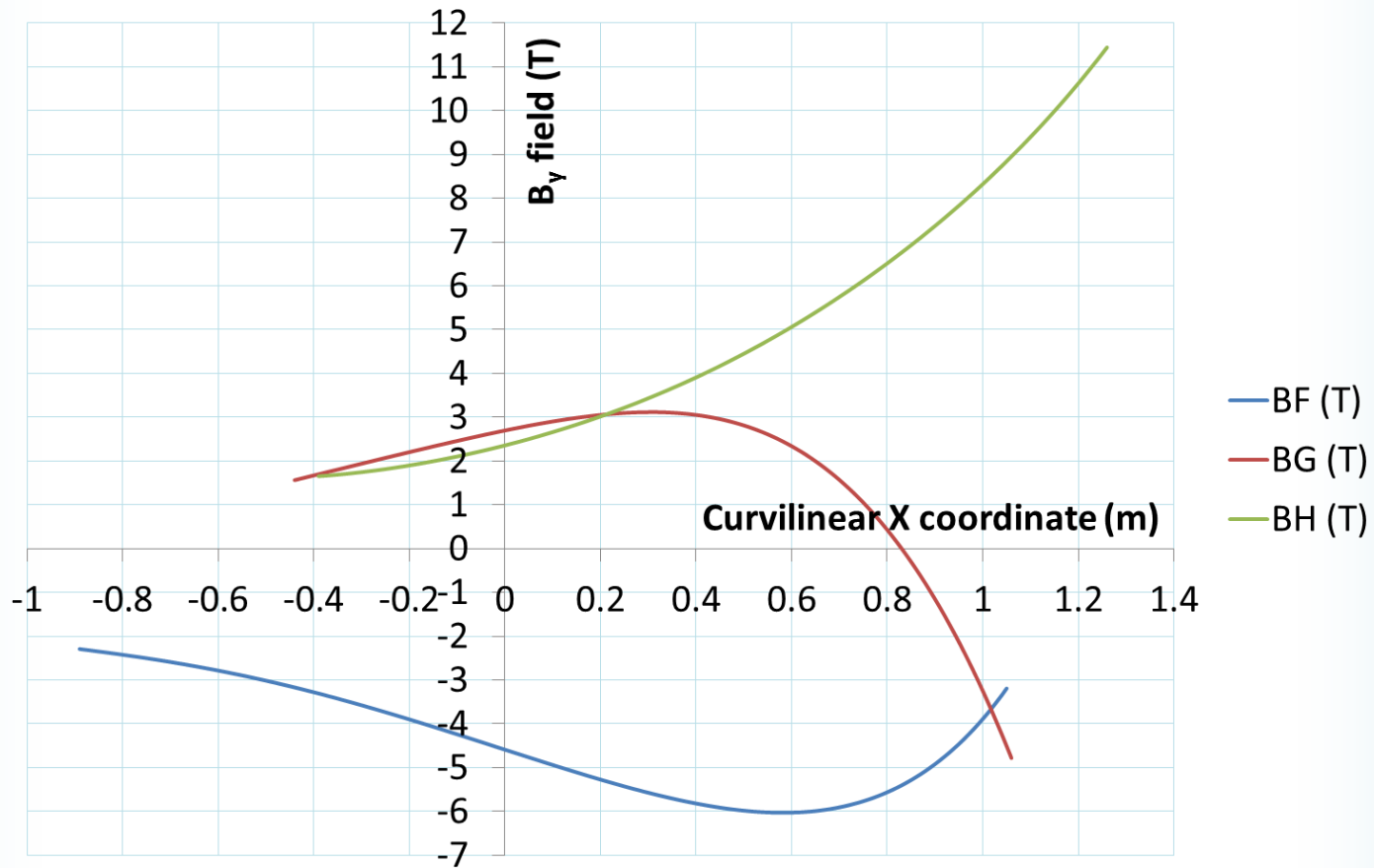
```

```
xnext[0] = -0.38443
xnext[1] = -0.12095
xnext[2] = 0
xnext[3] = 0
```

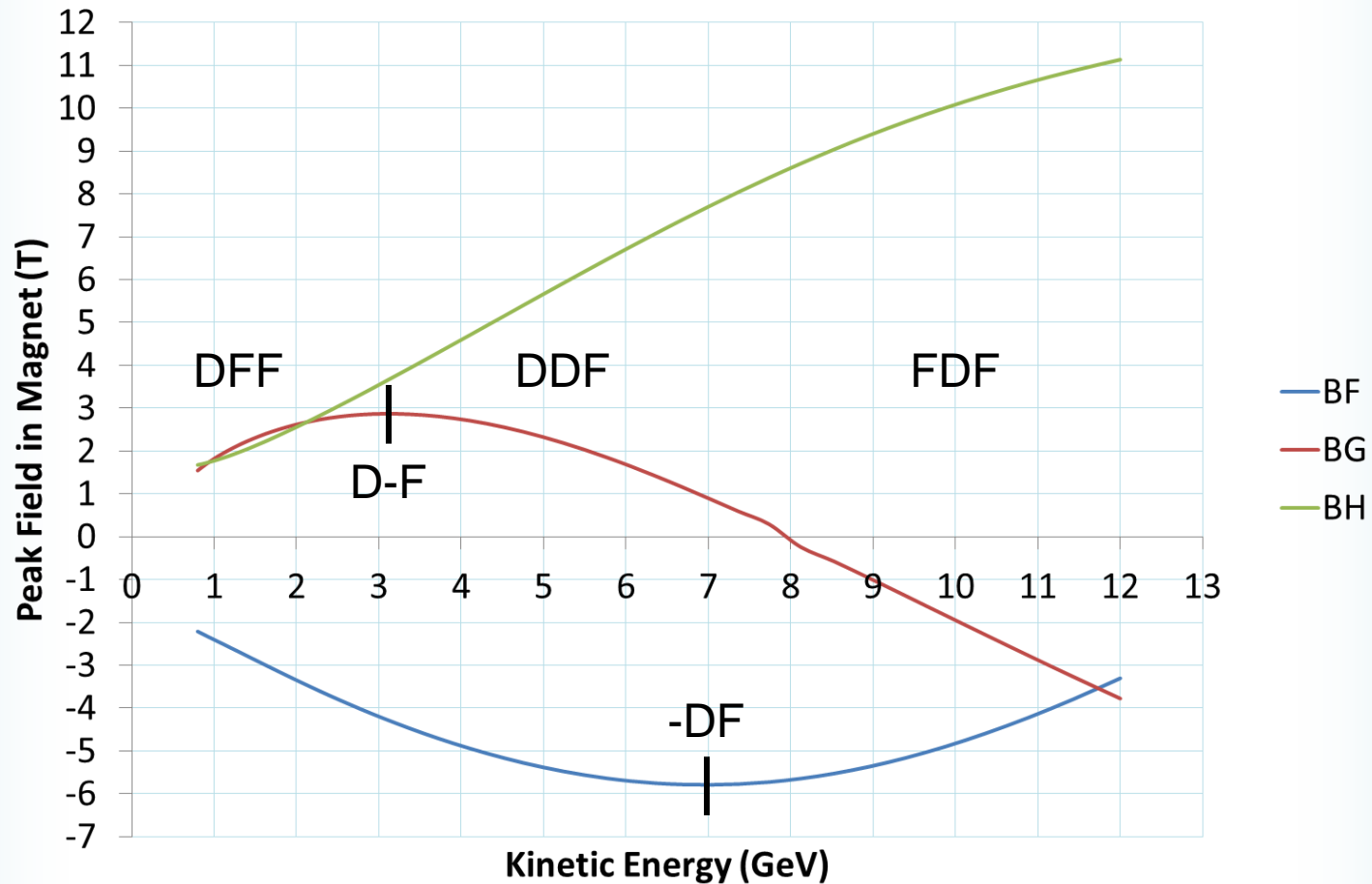
```
dd=8.70404e-01  
lastdd=1.9170  
steamul=1
```

Press I

Magnet Field Profiles



Magnetic Lattice with Energy



Well that was fun but so what?

- Door opened to non-scaling FFAGs with wide momentum range and properly fixed tunes
 - Either via global optimisation
 - Could apply to FFAG gantry or insertion problems
 - Or Grahame Rees's integration technique using the chromaticity (solve $dQ_{x,y}/dp = 0$ for B''_n)
- This “proton driver” design needs more magnets bending the right way at 12GeV
 - Advance the pseudophase?

V. Proton Omni-Ring

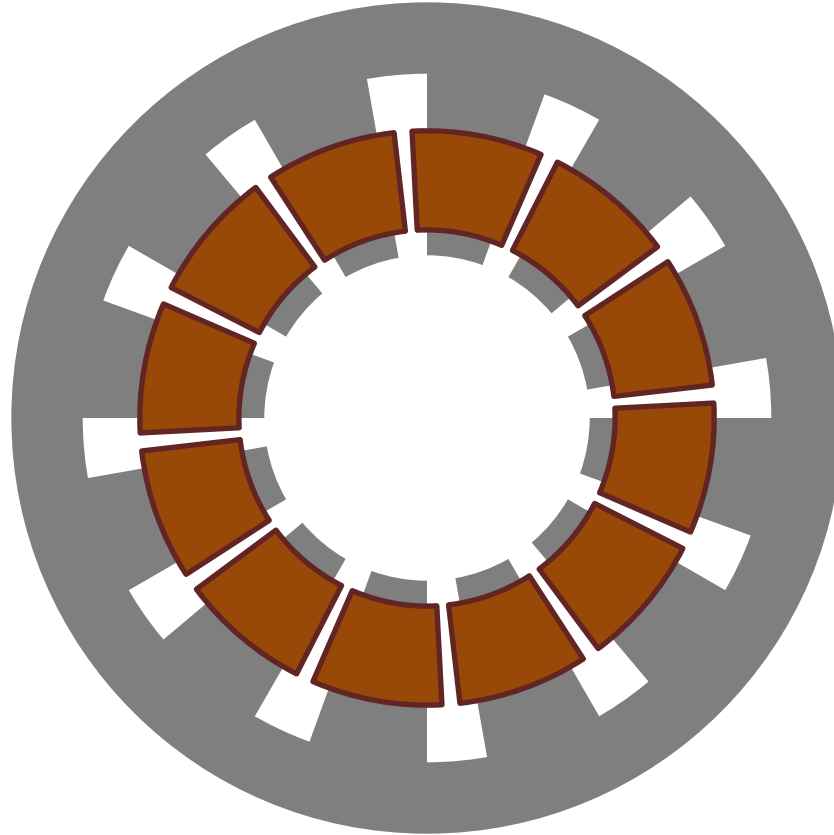
Configurable Proton R&D Ring

- Magnets with independently-powered coils can provide nearly arbitrary combinations of multipoles up to a certain order
- May be used to make a general-purpose FFAG and synchrotron test ring for beam dynamics studies, if apertures reasonably large
 - Good fit for FETS, 3MeV, H^- , space in R9 (RAL)
- Normal-conducting, simulated with Poisson

Possible Parameters

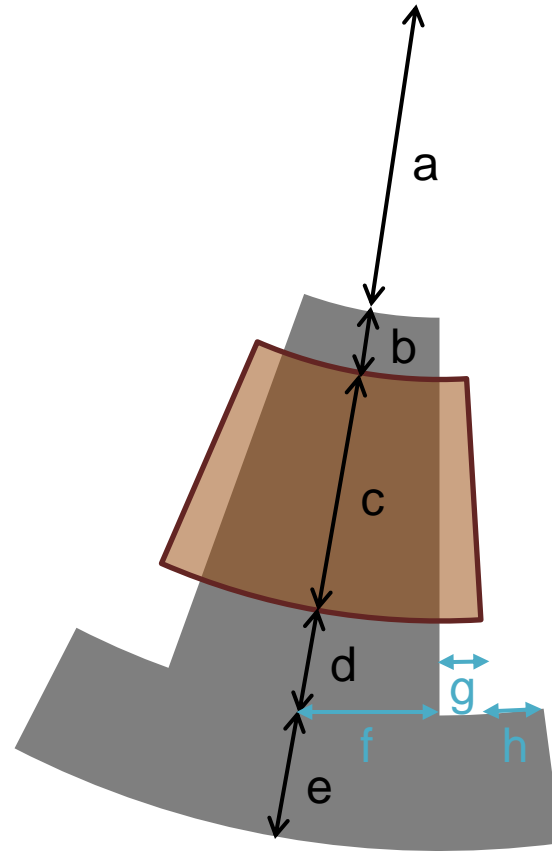
- Note: $3\text{MeV} = 75.1 \text{ MeV}/c$ for protons/ H^-
 - 4x as hard to bend as EMMA electrons already
- 0.2T dipole at 40% packing \rightarrow 6.3m diameter
 - Compare EMMA at 5.3m
 - 24 magnets \rightarrow 33cm magnet, 49cm drift per cell
 - Fits in R9, can branch off from $>3\text{MeV}$ linac test stand (CH structure tanks etc.)
- Test: space charge, injection, FFAGs, halo...

Omni-Ring Magnet



- Dodecapole with separately-powered coils
- Calibrated to produce multipole fields

Geometry Parameters (1/12 magnet)

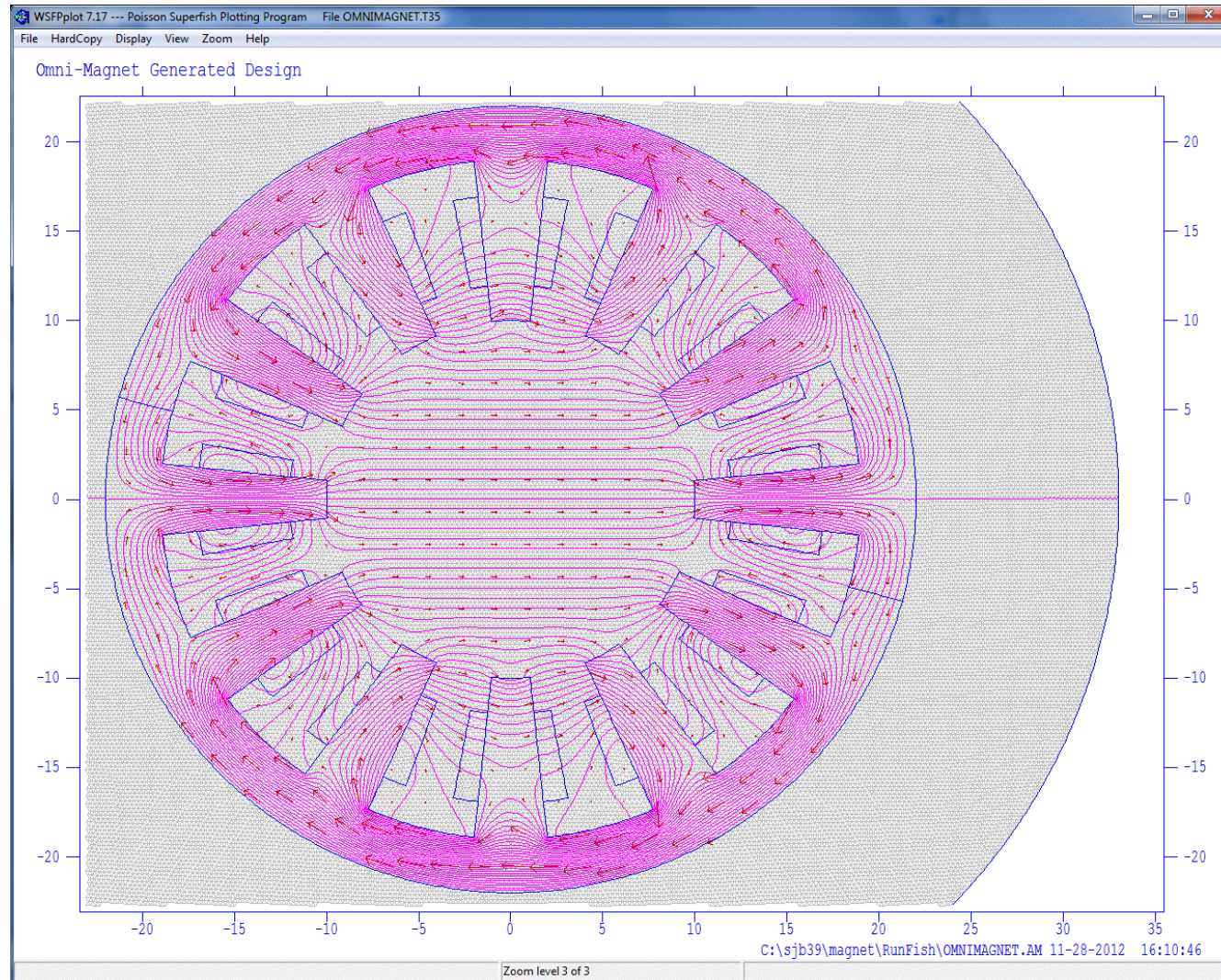


- Aperture = $2a$, coil thickness = c , yoke = e , etc.
- f =pole fraction, g =coil fraction, $f+g+h=1$

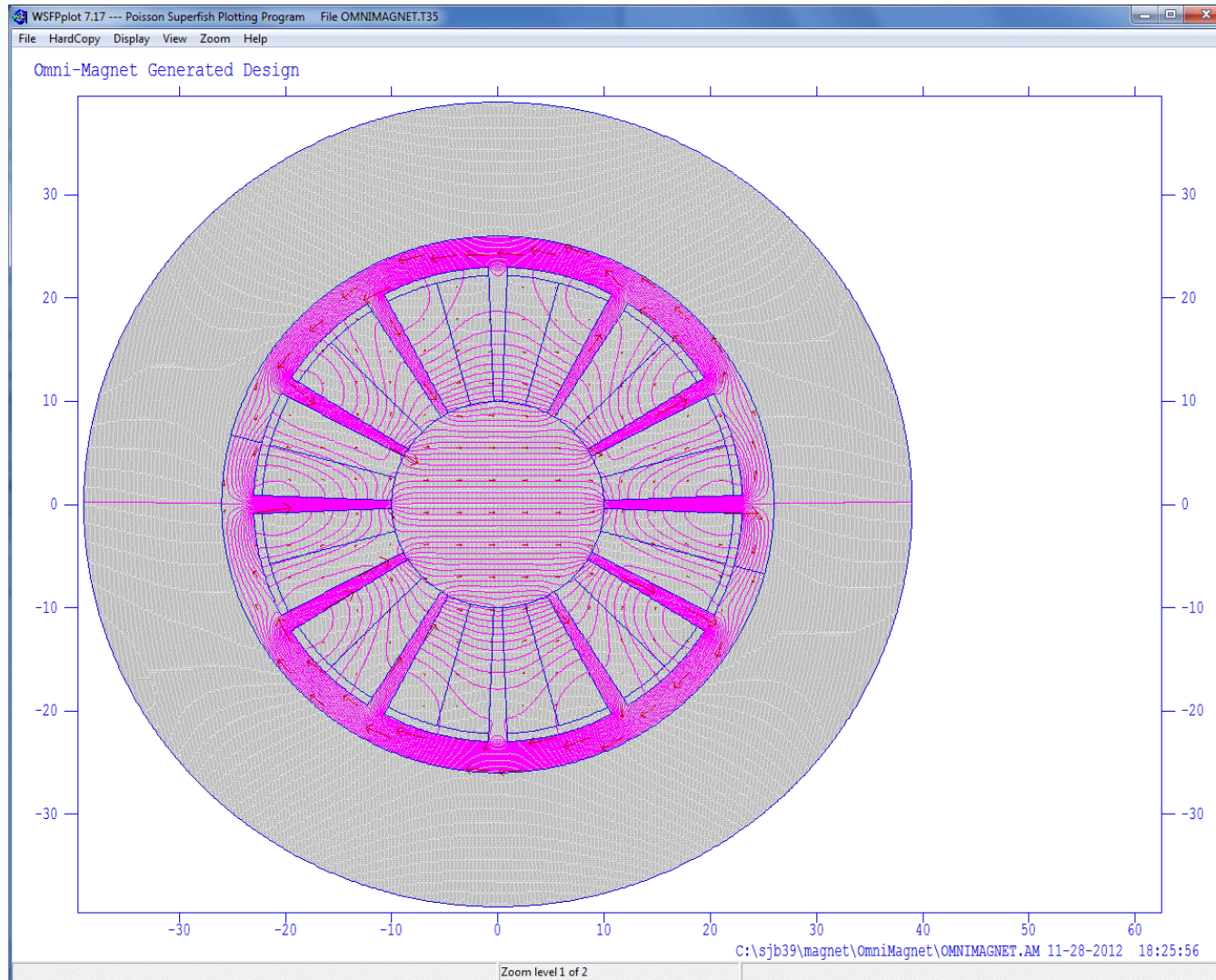
Compare ISIS EPB2 magnet “Q11”

- 235A in 10x10mm coils
 - **2.35A/mm²** in coil+water+insulator overall
- 5.4kW total power (water-cooled copper)
- 105mm radius physical aperture
 - **80mm radius good field $\pm 0.5\%$**
- $3.76\text{T/m} * 105\text{mm} = 0.395\text{T}$ pole tip field
 - **Spec says up to 1.4T flux in return arms**

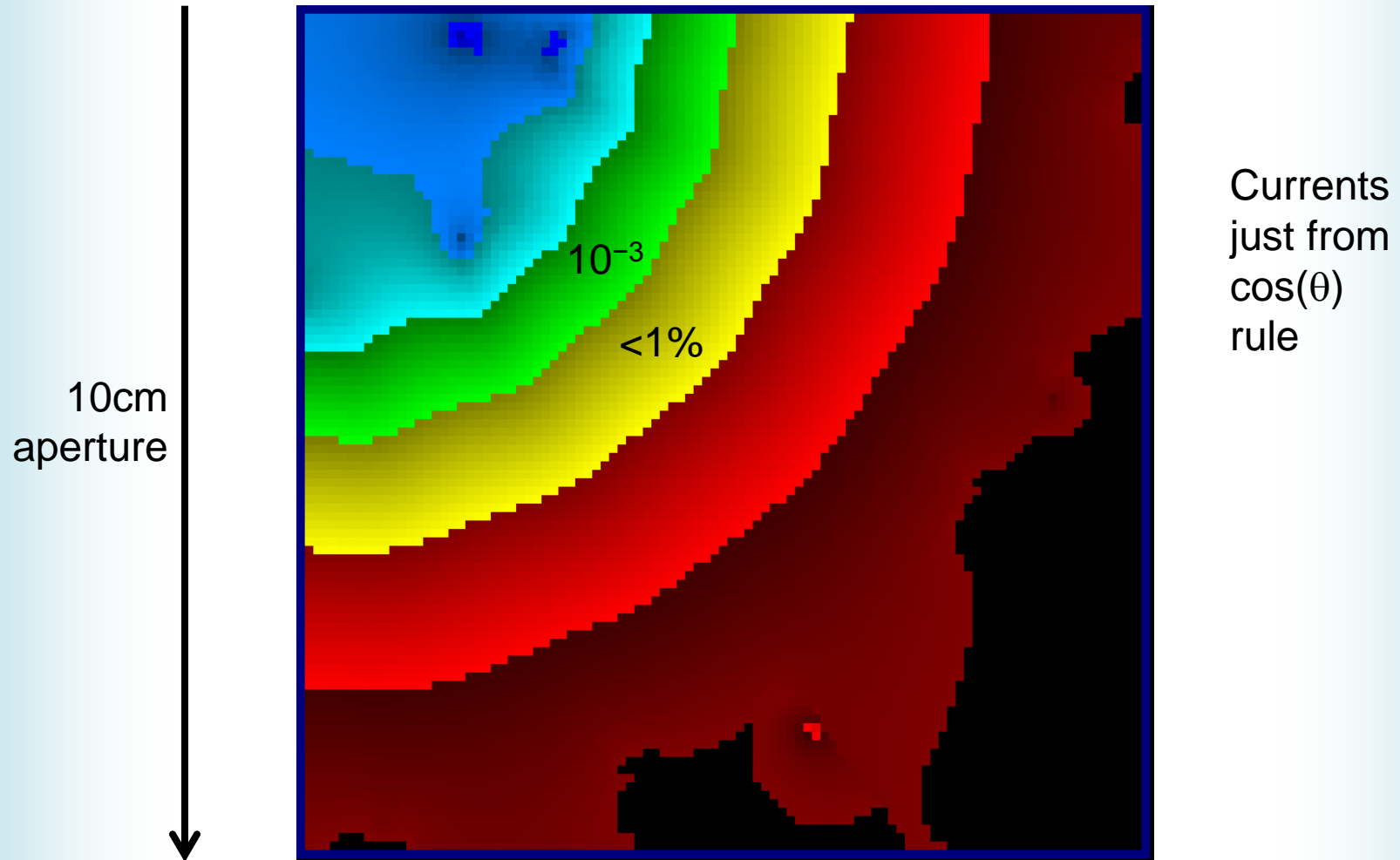
Before Optimisation (0.0158 T)



After Optimisation (0.1141 T)



Field Quality (Dipole case)



Magnet Practicalities

- Neil Marks used this idea for the corrector magnets of SRS at Daresbury Laboratory (UK)!
 - Could actually be used on ISIS too
- Can it be done cheaply/practically?
 - Needs many-channel power supply
 - Maybe current density can go higher?
- Calibration is an interesting problem
 - Use standardised test rig for all magnets
 - Integrate magnet field sensors into poles?

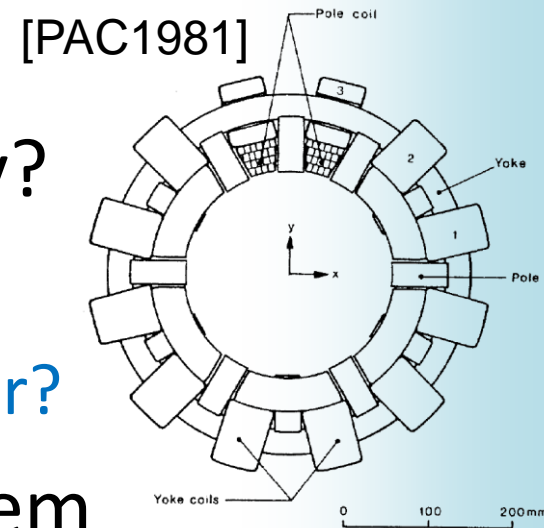


Fig.1 Cross-section of the multipole magnet



Other Ideas & Issues

- Ring can test chopping, injection painting, variable space charge levels, arbitrary tune settings, magnet nonlinearities
- Could do sequential single-turn extractions
 - Beam “movie”
 - Purpose-built halo diagnostic in extraction line?
- “Optical bench” positioning setup, bellows??
- Realisation of cheap ring at 3MeV hinges on whether H^- stripping possible (foils too thin?)

The End: FFLAG Master Table

2013 Update

Table 1: Classification of FFLAGs and their characteristics. Uppercase ‘Y’ indicates property is always true, lowercase ‘y’ that it is achievable in some cases. ‘3+’ means three or more lenses per cell are required.

Type of FFLAG	Fixed tunes	Wide E range	Isochronous	Small ring
Scaling	Y	Y	N	N
Non-scaling	3+ 	y	y†	y†
Linear n.s.	N	N	y(quasi)	y
Vertical s.	Y	Y	N	N
V. n.s.	3+	?	?	?
Linear v.n.s.‡	?	?	?	?
Skew	y	y	y 	?

...but spiral better

† Two ‘y’s may not be achievable simultaneously.

‡ Linear field VFFAG suggested by D.J. Kelliher.