Ultra-low emittance beams from ion traps for high precision collisions

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PROJECT NARRATIVE

1.0 Background/Introduction

The scientific output of a collider or any accelerator-based facility is dependent on producing certain particle interactions more densely than found in nature. This density can be achieved either by adding more particles (higher beam power) or putting them in a smaller space (cold beams). This proposal examines one way of producing ultra-cold beams (but likely with fewer particles), in conjunction with a strong focal point, which may be a future route to generating particle interactions with more energy efficiency. It also appears to allow entirely new kinds of high-density, low-temperature collisions of relevance to nuclear physics. The following subsections will help describe this more precisely.

1.1 Emittance

The volume in position-momentum phase space of a collection of particles is generally conserved, by Liouville's theorem, with exceptions only if the force depends on particle velocity (dissipative force) or can vary on such a small scale that individual particles can be resolved (as in stochastic cooling of beams [1]). This phase space volume is a useful measure of the "size" of a beam since it will remain unchanged through most electromagnetic beamlines. If the X/Y/Z dynamics are decoupled, area in a single phase space plane such as (x,p_x) can be considered. Beam distributions often have long tails, so RMS quantities can be used instead of 100% areas, such as $\sigma_x\sigma_p$ for a distribution where x and p are uncorrelated, or $\sqrt{\sigma_x^2\sigma_p^2 - \mathrm{cov}(x,p)^2}$ otherwise. Dividing this (by convention) by the particle rest mass m_0 and the speed of light gives a quantity known as *normalized RMS emittance*, which for uncorrelated x and p is

$$\varepsilon_{\text{norm,rms}} = \frac{\sigma_x\sigma_p}{m_0 c} = \sigma_x\sigma_{\beta\gamma} \cong \frac{\sigma_x\sigma_v}{c}.$$

Here, $\beta=v/c$ and $\gamma=(1-\beta^2)^{-1/2}$ are the relativistic factors and $p=m_0c\beta\gamma$ has been used. The final term uses the $\gamma=1$ nonrelativistic approximation. Typical ion sources for accelerators [2] produce $\varepsilon_{\text{norm,rms}}$ in the 10^{-7}–10^{-6} m range, whereas laser Doppler cooled ion traps [3] produce 10^{-13}–10^{-12} m. Additional cooling techniques [4] have achieved on the order of the single particle limit $\varepsilon_{\text{norm,rms}}=\hbar/(2m_0c)$, dictated by the Heisenberg uncertainty relation $\sigma_x\sigma_p \geq \hbar/2$. For a $^{40}\text{Ca}^+$ ion commonly used in laser Doppler cooling, this is 5.26×10^{-17} m. It is clear from these values that there is a vast parameter space of ultra-low emittance beams.

1.2 Luminosity and Efficiency

Particle interactions require particles to "hit" each other and the ease with which this happens is quantified by an area known as the *cross section* (σ, not to be confused with beam size). A colliding beam focal point has a *luminosity* L defined such that the rate of the interactions is Lσ. Common units for L are $\text{cm}^{-2}\text{s}^{-1}$. A common relation between beam parameters and luminosity for a collider is given by eqn. (16) in [5]:

$$L = \frac{N_1 N_2 f N_b}{4\pi\sigma_x\sigma_y},$$

where N_1 and N_2 are the numbers of particles per bunch, and fN_b is the rate of bunch crossings (N_b bunches in a ring of revolution frequency f). σ_x and σ_y are the Gaussian beam sizes at the focus. Clearly, smaller beam sizes will give more luminosity and fewer particles per bunch will give less. The scaling where $\sigma_{x,y} \propto N_{1,2}$ is interesting because L is constant while the number of particles required can be reduced. This is a path to *high specific luminosity*; that is, luminosity per unit beam power.

If the RMS opening angle of the focal point is θ, the beam sizes are related to the normalized RMS emittance via $\sigma_{x,y}=\varepsilon_{\text{norm,rms}}/(\beta\gamma\theta)$, where β and γ are again the relativistic factors. The maximum practical opening angle is dictated by the detector length and feasible magnet apertures, so consider it to be constant for now, along with the beam energy. This means the constant-luminosity scaling law becomes $\sigma_{x,y} \propto \varepsilon_{\text{norm,rms}} \propto N$.

That is, smaller emittances such as those produced by cooled ion traps are a path to achieving the same luminosity with fewer particles, meaning lower power consumption and less activation of the accelerator.

Just as the emittance had a large scope for reduction (a factor of 10^6 or more), an example from an operating collider can show the potential efficiency increase. The Relativistic Heavy Ion Collider (RHIC) Run 15 collided two beams of polarized protons, each at 100 GeV kinetic energy. This run achieved [6] a peak luminosity of $L=1.28\times10^{32}$ cm^{-2}s^{-1} with $N=1.92\times10^{11}$ protons in each bunch. The total cross section of proton-proton collisions at a total energy of 200GeV [7] is $\sigma_{tot}=51.81$ mbarns$=5.18\times10^{-26}$ cm^2, leading to an estimated interaction rate of $L\sigma=6.63$ MHz. However, the average bunch crossing rate was 8.68 MHz, implying an average 0.764 interactions per bunch crossing, less than 10^{-11} of the total population of the bunch. To put it another way, the circulating beam current was 0.267 Amps but the interacting beam current was only 1.06 pA. The luminosity together with the focal point setup implies $\varepsilon_{norm,rms}=2.52\times10^{-6}$ m, so we can consider applying the constant-luminosity scaling law to reduce ε and N by a factor of 10^7 to values more typical of laser-cooled ion traps: $\varepsilon_{norm,rms}=2.52\times10^{-13}$ m and $N=19200$. The same luminosity is produced but with a circulating current of only 26.7 nA, which would certainly reduce any high-current problems (space charge, impedances) in the machine.

What are the potential pitfalls of this extreme approach? One is that the beam size at the interaction point $\sigma_{x,y}$ also reduces by the same factor as the emittance: in this case from 0.14 mm to 0.014 nm, which would require very tight alignment and mitigation of vibrations. Another is that the beam would "burn off" in about 25,000 turns or 0.32 seconds, requiring rapid refilling of the machine, although this is an inevitable consequence of demanding the same number of interactions from fewer particles. Linear colliders and rapidly refilling fixed-field accelerator (FFA) schemes exist that could do this. The ultra-low emittance bunch itself would have to survive from the source to the collision point without undergoing emittance growth or random jitter, requiring highly stable beamlines and acceleration hardware. The positive potential is that with the accelerator no longer limited by high-current effects (and with a factor of 10^7 headroom), the beam current and bunch repetition rate could be greatly increased along with the luminosity.

The discussion up to this point has not mentioned the longitudinal axis, beam-beam tune shift [8] or intra-beam scattering (IBS) [9]. However, the constant-luminosity scaling still works considering that the fully general formula for integrated luminosity per bunch crossing integrates the product of two particle density functions (each with units m^{-3}) over space and time (units m^{3}s) multiplied by relative speed (units m/s). The result has units m^{-2} as expected, but if the total number of particles and physical size are both reduced 10 times, then both densities increase 100-fold and the overlap in space-time decreases by 10^4, yielding constant integrated luminosity. The only condition is that the longitudinal emittance and bunch length at the focus also become smaller proportional to N. Fortunately, ion traps provide small emittances in all three planes and an energy chirp can be used for longitudinal compression. Effects such as beam-beam tune shift and IBS are also left constant by this scaling, mainly because going 10 times closer to a charge 10 times smaller results in the same angular deflection. The linear beam-beam parameter from eqn. (17) in [8] is given by $\xi = \frac{N r_0 \beta^*}{4\pi\gamma\sigma^2}$, where $\beta^*=\sigma/\theta$ decreases in proportion to N during the scaling, canceling the variation of σ^2 on the denominator, leaving everything else a constant. To mitigate IBS, it is preferred to leave the ultra-low emittance beam physically large but very sparse in all places except the focus.

The availability of an ultra-low emittance beam source would enable exploration of this large, mainly unexplored parameter space of high specific luminosity colliders. There will also have to be development of highly stable acceleration beamlines that preserve the small emittance, but a problem of high-current effects (dictated by physics) has now been converted into a problem of equipment precision (dictated by engineering). Extreme examples of precise alignment have been devised for gravitational wave observatories [10], giving hope for future developments in this direction.

1.3 Ion Traps

Laser-cooled ion traps have found much use in atomic and optical physics for manipulating the states of individual ions; for instance, [11] gives a thorough account of the dynamics of such a trap, cooling of the ions, and its use in quantum computing. The IBEX experiment [12] at Rutherford Appleton Laboratory (RAL) uses a non-cooled ion trap to emulate beam bunches under space charge conditions. The S-POD trap at the University of Hiroshima [3] has extracted small populations of cooled ions from such a trap to form nanobeams as well as accumulating 0.8×10^7 ions in a single cooled trap [13].

These traps all have the linear Paul trap geometry, where DC electrodes provide longitudinal confinement, while four rod-shaped RF electrodes provide transverse confinement. The RF is required because $\nabla\cdot\mathbf{E}=\rho/\varepsilon_0$ implies there is no DC electrode configuration that will produce a 3D potential minimum in a vacuum. Instead, effective transverse confinement is produced in the same way that alternating gradient focusing works in accelerator beamlines: Alternating signs of electrostatic quadrupole lenses with delays between them produce overall focusing in both transverse directions. There are other geometries possible, such as a higher-order transverse multipole (IBEX has a weaker configurable octupole superimposed on the transverse quadrupole), and the trap may be broken into many longitudinal segments separated by potential barriers, which is used in IBEX to move the ions from one trap potential to another. Extraction can be performed by lowering the final potential barrier, which will accelerate the ions out of the trap, such as onto a diagnostic screen (as in IBEX) or potentially into a more developed transport beamline. This requires there to be a hole in the longitudinal capping electrode large enough for the bunch to go through.

The ions can be produced by several methods. An electron gun firing a beam through the trap volume is used in IBEX to ionize millions of argon atoms. Other systems, such as the Oxford quantum computing group's trap, use two-stage laser excitation to be selective on atom species and potentially velocity via the Doppler effect. The gaseous atoms can be provided either via a gas valve or an atomic effusive oven source, both of which are simple to implement.

Once the ions are inside the trap, they are subject to a balance of the trap's focusing force and the space-charge repulsion of the same-charge ions, which forms a limit on the density. Considering just the x axis, if the trapping force is linear $F=-kx$ and the Coulomb repulsive force is $F=(1/4\pi\varepsilon_0)qQ/x^2$ at the edge of a spherical bunch of charge Q, then setting the total force to zero requires $x^3 \propto Q$. x^3 is proportional to volume, so this says the trap can accumulate additional ions at a constant 3D charge density.

1.4 Laser Doppler Cooling

When a photon of wavevector $\mathbf{k}$ is absorbed by an electron bound to an atom or ion, it imparts a momentum kick of $\hbar\mathbf{k}$ and excites the electron to a higher level. This excited electron will eventually decay to a lower level and, assuming this is the same one, a photon of momentum magnitude $\hbar|\mathbf{k}|$ will be released in a random direction, giving the ion a kick in the opposite direction. The probability of absorbing a photon depends on its wavelength in the rest frame of the ion and how close this wavelength is to the center of the relevant absorption line. This means the average force on an ion in a laser beam will vary depending on the ion's velocity in the laser beam axis, via the Doppler shift of the laser. Note that the emitted photons produce an average force of zero because they are emitted in random directions. This mechanism is harnessed in Laser Doppler cooling, as a force that is dependent on velocity is non-Liouvillian (or dissipative, like friction) and can change the total energy of the ion. An analysis of this process is given in [14].

The cooling will often continue until the ions "solidify" into an ordered state known as a Coulomb crystal (see [3] for example), where forces from Coulomb repulsion are balanced by the trap confinement. Like in a solid with finite temperature, the ions may still have some residual vibration but do not structurally rearrange. Liquid and gas-like states are also possible at higher temperatures [15].

The lower limit of temperature using only Doppler cooling (assuming a 2-level atom and no other techniques such as sideband cooling [4]) is $T_D=\hbar\Gamma/2k_B$, where Γ is the absorption line width. This is dictated by the momentum kicks from the randomly emitted photons. Note that this temperature is in a single axis only, for instance $k_BT/2=\langle mv_x^2/2\rangle$, assuming non-relativistic speeds and using < > for the average over all particles. This gives an expression for the RMS velocity at the Doppler cooling limit $\sigma_v^2=k_BT_D/m$, which can be used to find the normalized RMS emittance in the trap at this limit

$$\varepsilon_{\text{norm,rms}} \cong \frac{\sigma_x\sigma_v}{c} = \frac{\sigma_x}{c}\sqrt{\frac{k_BT_D}{m}} = \frac{\sigma_x}{c}\sqrt{\frac{\hbar\Gamma}{2m}},$$

where σ_x is the RMS size of the trapped ions in this axis. For $^{40}\text{Ca}^+$ ions, with a linewidth of Γ=23 MHz×2π being used (from [11]) and σ_x=1 mm, this gives $\varepsilon_{\text{norm,rms}}=1.13\times10^{-12}$ m, already far lower than traditional ion sources for accelerators. This means a lot of interesting experiments can be done in the ultra-low emittance beam regime before requiring additional cooling methods such as sideband cooling, although these have been implemented elsewhere and are possible for a potential upgrade.

Implementation of laser Doppler cooling requires only that the lasers can reach the ions through windows in the vacuum chamber and gaps in the trapping electrodes. The laser intensity needs to reach the order of the saturation intensity of the transition, which in [11] is 48mW/cm^2, a comparatively low value. The more experimentally tricky aspect is that the laser frequency must be tunable with precision on the order of the absorption line width (MHz), which can be done using tunable diode lasers, an adjustable optical reference cavity, and a wavemeter (about $100k in all). The traditional geometry for cooling in 3D is "optical molasses," in which six laser beams come from both directions along all three axes. This is easily understandable as each axis resembles the 1D analysis in [14] and provides guaranteed cooling. If producing laser beams from all six directions is practically difficult, a single laser on the space diagonal will also usually work, as it couples to all three planes and on average extracts energy from each. The cooling rate will be somewhat slower and if a vibrational mode of the trap happens to be exactly perpendicular to the beam, it won't be cooled, but this can be avoided.

1.5 This Proposal: Ultra-Low Emittance Foci and Other Accelerator Applications

The ultra-low emittance of the cooled ion trap source suggests that extremely small focal points, with high specific luminosity, can be produced. This proposal will investigate this by adjoining a beamline to the extraction side of the ion trap, likely containing electrostatic lenses and acceleration. The beamline will end in a focal point, analogous to collider interaction points but scaled down in energy to a few keV.

With a low energy beam, focusing strength is easy to achieve and a comparatively large opening angle like θ=100 mrad can be used. The emittance alone predicts the minimum focal size via $\sigma_{x,y} \geq \varepsilon_{\text{norm,rms}}/(\beta\gamma\theta)$, where $\beta\gamma=7.33\times10^{-4}$ for $^{40}\text{Ca}^+$ accelerated through 10kV. Taking a value of $\varepsilon_{\text{norm,rms}}=9.02\times10^{-14}$ m from a recent simulation with N=500 ions, the emittance predicts an RMS focal size of 1.23 nm. However, emittance is no longer the only limit on focal size, particularly at low energy (or with high charge state ions). The Coulomb repulsion of ions of charge q can stop them at a radius of

$$r_{min} = \frac{1}{4\pi\epsilon_0}\frac{Nq^2}{m_0c^2\left(\sqrt{1+(\beta\gamma\theta)^2}-1\right)},$$

which is equal to 7.20 nm in this example for singly charged Ca^+. These focal sizes decrease with increasing energy (and $\beta\gamma$), so if the focus is small enough to push the Ca^+ ions within each other's electron orbitals, the +20e charge of the nucleus will no longer be well-shielded and q will gradually increase to this value.

Verifying that ultra-low emittance beams can be produced and utilized in this way, while confirming that the focal size obeys the expected scaling law with increasing energy, is a goal that sets the direction of this project. The techniques for doing so are likely to be important for future cold beam colliders.

The formation of dense foci in this scenario depends mainly on experimental precision rather than the power/energy/size of the facility, so a major thrust of this research is to test feedback and automatic aberration correction systems for forming the focal point. The scattered ions from the focus can be detected, so the plan is to use their outgoing angles to inform the alignment, an approach that works no matter how small the focus gets. Reducing the ion energy will increase the closest approach distance, so the alignment can be performed first at low energy with a comparatively large "target" to hit and then refined for successfully higher energies with smaller adjustments.

An ultra-low emittance ion source at Brookhaven National Laboratory (BNL) would also allow injection into the RHIC or future Electron-Ion Collider (EIC) hadron accelerator chain, for example, to precisely measure sources of emittance growth. The simulation codes and experimental platform of the ion trap will also allow testing of accelerator-relevant concepts, such as increasing the cooling rate or throughput of the ultra-cold source, via modified trap geometries and other optimizations. Sympathetic cooling [16] of other ion species will also be of interest.

1.6 Comparison to Other Techniques

Extraction out of a cooled ion trap has not been frequently explored for accelerator use, with the most advanced application appearing to be the S-POD trap at the University of Hiroshima [3], which produced nanobeams for ion implantation in diamond. Manipulation of Coulomb crystals in transfer lines to produce small dense foci appears so far untried.

Coulomb crystals have been produced in motion in low-velocity storage rings [17], which share many features with stationary Paul traps. The rings have more trapping locations for producing more crystals, but this can also be done with a linear trap by introducing many longitudinal sections. This suggests another configuration that may be a large improvement in throughput for future cold beam colliders: a linear cooling channel, where uncooled ions continuously enter at the start and Coulomb crystals emerge from the end. The ion trap volume is only a few mm in size, so to match the RHIC average bunch collision rate of 8.68 MHz with 5 mm between crystals would require a velocity of 43.4 km/s (compared with the 2.8 km/s in [17]). The cooling process takes ~10 ms so the channel length would be ~434 m, which could be coiled up: a helix instead of a ring, with the volume immersed in laser radiation (not too difficult given the low intensities required). A stationary trap, by comparison, could cycle at most at ~100 Hz: nearly 10^5 times slower. This proposal does not have the resources to construct such a long channel but precursor experiments that inform its design may be possible using the hardware.

Conventional cooling systems in accelerators work at high energy (many MeV or GeV) but generally have slow time constants of minutes or hours. A common theme when examining cooling systems is that the slow rate is a "bottleneck" for cooled beam current. If the cooling is active at the same time as an emittance growth process (like beam-beam or intra-beam scattering), the equilibrium emittance is the best that can be achieved. If full cooling without competing emittance growth is required, a long time must be devoted to cooling before the main facility gets a single pulse of fully cooled beam (for instance, the International Linear Collider (ILC) damping rings). Mitigating these problems for higher throughput, and finding yet lower emittances, is the inspiration for the linear laser cooling channel described above.

Relevance to the Mission of Nuclear Physics (NP)

The Nuclear Physics topical area "Accelerator Research and Development for Current and Future Nuclear Physics Facilities" in the funding call states:

- Research aimed at transformative advances in ion sources […] and beam cooling is also encouraged.

This fits with the description of the laser-cooled ion trap in previous sections.

Further relevance to the Nuclear Physics mission goals can be seen by examining how an ultra-low emittance focus would work in a future facility operating at higher beam energies. The focal size continues to decrease with increasing energy, regardless of whether limited by emittance or Coulomb repulsion. The density of the focus can be calculated approximately via

$$\rho = M/V = Nm_0/((4/3)\pi r_{min}^3),$$

where m_0 is the ion mass and M and V are the total bunch mass and volume. Densities from this formula are given in Table 1, where the first row may be achievable in this early career award project. One goal will be to measure the beginning of the steep scaling of density with bunch kinetic energy (E_k).

Table 1. Parameter sets for Coulomb-repulsion-limited ion bunch foci. Z=q/e is ion charge number.

Ion Species	Ion population	E_k/Z	r_{min} (Coulomb)	ρ/ρ_{water}	Min. normalized emittance required (m)
$^{40}Ca^+$	**20000**	**100 kV**	**28.8 nm**	**0.0133 (dense gas)**	**6.67e-12**
$^{40}Ca^+$	20000	1 MV	2.88 nm	13.3 (~Hg)	2.11e-12
$^{40}Ca^+$	20000	10 MV	0.288 nm	13300 (white dwarf)	6.67e-13
$^{40}Ca^{20+}$	23700	83 GV	0.1 pm	3.75e14 (nuclear density)	4.56e-13
$^3H^+$	120	2 GV	6.39 fm	5.46e14	8.91e-16

Table 1 assumes the bunch is focussed with a 100 mrad opening angle and a $\delta p/p=10\%$ momentum spread chirp, leading to an implosion energy of about 1% of the mean forward kinetic energy. The rightmost column keeps track of the maximum 1-plane source emittance $\varepsilon_{norm,rms} \leq \beta\gamma r_{min}\theta$ required to make the focus Coulomb-limited (note that 1e-13 is readily achievable [3]). By row 3, the ions are starting to penetrate each other's electron shells, so charge shielding is reduced and line 4 uses Ca^{20+}.

Foci approaching the density of white dwarf matter (degenerate matter) can be formed if the ultra-low emittance focus can be obtained at 10 MeV energies. This happens from the tight focus of a single bunch: no colliding bunch is required, although any number of colliding bunches may be added because the position of the focus is spatially controlled with the alignment required to produce the focus.

Lines 4 and 5 extrapolate to possible future facilities at high energy. Line 4 makes a droplet of neutron matter, containing the mass of a few thousand ions, using a beam energy of 83GeV per charge and state-of-the-art 0.1 pm alignment such as used for stabilizing the LIGO mirrors [10]. This collision environment would also allow for the study of stellar nucleosynthesis paths, such as the r-process that only happens at high neutron densities, like those found in supernovae or neutron stars.

The ability to collide multiple nuclei at once, rather than pairwise as in present colliders, opens an alternate path to making neutron-rich superheavy nuclei that may be comparatively stable. Line 5 performs multi-way fusion from tritium to make a neutron-rich superheavy element (requiring 16 times better alignment) at slightly lower energy. The small emittance here can likely be achieved by sideband cooling and keeping the tritium ions in individual traps. The formula for r_{min} ensures the Coulomb barrier of assembling the nucleons into this small space is overcome, and the de Broglie wavelength of tritium is 0.318 fm at 2GeV, small enough that diffraction is not a concern.

An (ambitious) future facility based on this principle would advance the Nuclear Physics priority areas of:

- Understanding the limits of nuclear existence in nature;
- Understanding how heavy nuclei have emerged since the origin of the Universe and continue to be created via nucleo-synthesis in cataclysmic cosmic events; and
- Searching for undiscovered forms of nuclear matter.

In a broader perspective, this may be seen as part of the long-term evolution of physics experiments toward ever increasing degrees of control over matter. Since the 1980s, atoms have been able to be placed individually in deliberate locations and it is natural to wonder what the equivalent technology for nucleons would be. It must happen at high energy (~GeV or above) in order to provide spatial confinement against the uncertainty principle at the focus, but in principle custom nuclei could be assembled. The ultra-cold beam source may also be seen as eliminating entropy (temperature) from the experiment: A finite temperature implies there is something random about the initial state of the particles that the experimenter does not know. It is no coincidence that this is the same basic technology underlying quantum computers, which require the ions to be in a well-defined ground state. It will be a natural evolution of ultra-cold beam technology to use quantum degrees of freedom in the beams, which may eventually give greater tunability over the output from collisions. Proposals to use these cold beams in quantum computers are already under development [18].

There are also applications for machine learning in the algorithm that will use the detected scattered beam distribution to optimize the electromagnetic lenses leading up to the ultra-low emittance focus. Developing this algorithm, which will probably have several implementations, is an important part of this proposal.

2.0 Project Objectives

The main objectives of this project are:

- Extract an ultra-low emittance bunch from a laser-cooled ion trap and use a beamline to focus it.
- Measure the dependence of focal size on beam energy and compare it to theory.
- Test concepts for accelerator-relevant ultra-cold ion sources with increased current or different ion species, including consideration of injection into the RHIC/EIC accelerator chain.

Prerequisite for these is a sequence of smaller experimental steps:

1. Produce, trap, and detect ions;
2. Add laser and cool ions to ultra-low emittance;
3. Extract ions into beamline and measure distribution;
4. Accelerate and focus ultra-low emittance ion bunch;
5. Optimize ion focal point.

Of these, 1 through 3 have been done elsewhere, while 4 and 5 are novel to this proposal.

Scientific and/or Technical Merit of the Project

Much of the innovation of this proposal comes from the combination of laser-cooled ion traps normally used in atomic and optical physics with accelerator technology: a low energy beamline and focal point. This has not been tested before and is an important step in seeing if the benefits of ultra-cold ion technology can be transferred to higher energy beams and colliders.

It is also proposed to use the scattered ion distribution as a diagnostic to fine-tune the focal point and remove aberrations. This is potentially mathematically complex but has the benefit that the "signal" is always sensitive on the relevant distance scale: the size of the focus. This also provides a future solution to the problem of how to have positional feedback on scales smaller than atoms, and eventually towards the size of nuclei, as the focus gets smaller at many GeV.

Success is not guaranteed but it is hoped, at worst, to determine the primary error source affecting the formation of the ion focus. This can be done by increasing various sources of alignment error artificially and examining the sensitivities. In this case it can also be verified that the algorithm for tuning the focal point can always produce the focus down to the size limited by the errors (e.g., vibration) as the error amplitude is varied. This sort of analysis will also identify the primary error sources and suggest improvements (vibration isolation, active feedback, interferometric position measurements, etc.) to the equipment that may be possible within the five-year project. So, there is a case for cautious optimism. Additionally, any attempt to bring a Coulomb crystal out of equilibrium and then implode it to minimum size will be an experimental first and noteworthy in its own right. The algorithm to find and optimize the focus will be of general interest to collider projects.

The objective of using the cooled ion trap to test accelerator-relevant concepts may also produce serendipitous discoveries, especially considering the ultra-low emittance regime spans several orders of magnitude and is not well-explored so far. The addition of a configurable beamline to the ion trap will allow different optics concepts to be studied. With low-energy ions, the trap electrodes and beamline lenses will typically be cheaper than other components, such as the vacuum system and cooling laser. This means several different geometries could be tested on the same platform, both for the ion trap or the beamline.

A positive result, achieving close to the theoretically predicted focal sizes, would suggest that even smaller foci are possible in future follow-on projects at higher energies. These would start to compete with high density matter experiments using lasers. Eventually, the particle accelerator will provide more implosive energy to each particle than a laser (or future accelerators become laser-driven, in which case the point is moot). The beamline technology for preserving ultra-low emittance beams may differ from conventional accelerator components because of the increased stability requirements, but this work will clarify the pathway toward both higher density collisions and higher specific luminosity colliders. New ideas are required for future nuclear physics and high energy physics facilities to enter new regimes and produce significant discoveries. It is hoped that the study of ultra-low emittance beams will provide some clues.

Other research on cooling has often concentrated on cooling a beam in a storage ring or collider. This can be worthwhile for many applications, but cooling times are typically long, meaning that the cooling process either limits the current throughput of the machine or limits the minimum achievable emittance, being in constant competition with beam scattering processes. Cooling at the source and trying to preserve the emittance may seem a more difficult approach, but the ion trap source is a low-energy device, not an entire storage ring. This means many could be put in parallel at less expense to increase throughput (or the idea of a linear cooling channel as discussed earlier). Cooling at lower energy can also increase the energy efficiency compared to other techniques. For instance, synchrotron radiation damping requires particles to lose a multiple of their initial energy, which is replenished by RF acceleration, in order to achieve their equilibrium emittance.

3.0 Proposed Research and Methods

The first core piece of apparatus needed is the laser Doppler cooled ion trap, shown in Figures 1 and 2. It is a compact piece of equipment that has been built elsewhere [3,4,12] but this proposed project will involve adding a custom beamline and there may be frequent modifications to the trap for experiments, so the time and expenditure to build one at BNL merits the cost.

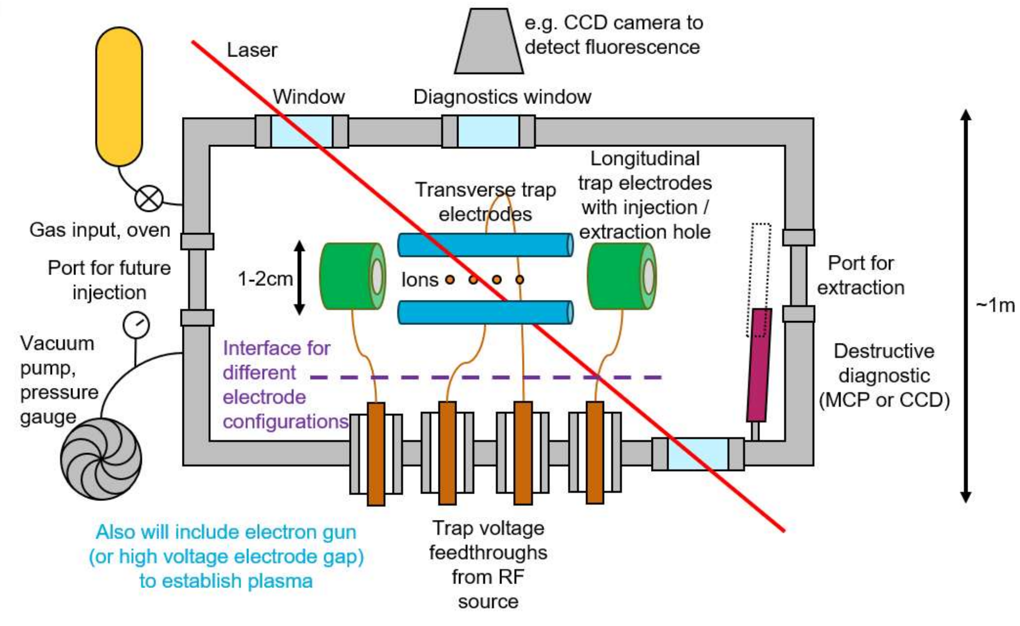

Figure 1. Schematic of a laser-cooled ion trap (Paul trap configuration).

The cooled ion trap itself is a flexible source with variable parameters over a wide range:

- Bunch charge can be changed via varying ion gas pressure and trap voltage.
- Bunch size can be controlled via trap voltage and collimation. The directional electrode voltages can also change the shape of the bunch from “cigar” or chain to “pancake” as well as spherical.

- Emittance and temperature can be set to a wide range of values by stopping the cooling process part way.
- Ion species other than the coolable ion (e.g., $^{40}Ca^+$) can be cooled using sympathetic cooling: mixing the desired species in contact with the coolable ion in the trap.
- Trap topology can be changed via the easily exchangeable electrode configuration. This includes a dipole+sextupole configuration with two stable points, for producing two bunches at once that may be separated and used in a collider topology, while still maintaining timing accuracy.

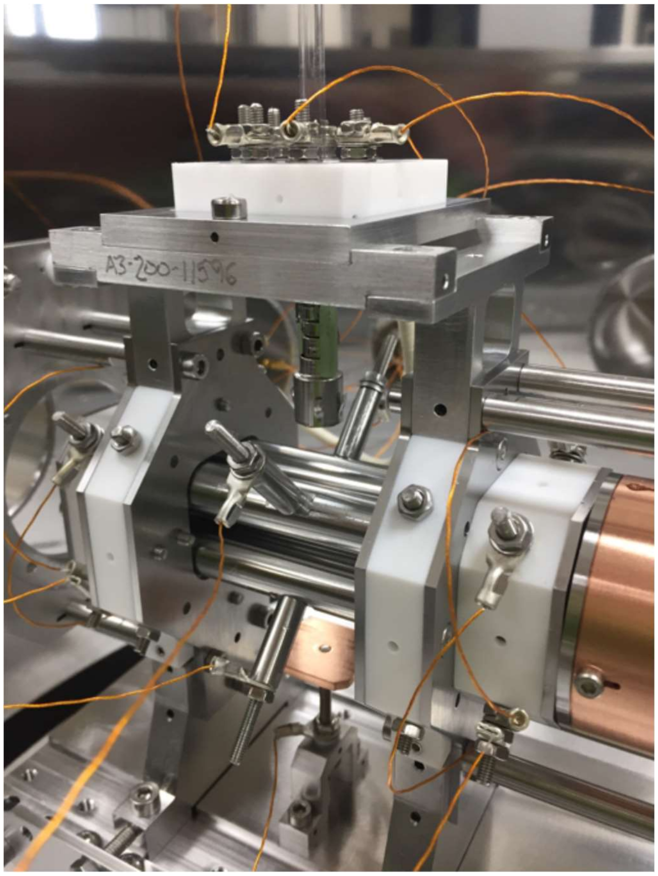

Figure 2 (left). The IBEX Paul trap [12] with vacuum chamber opened. The four rods in the center of the image are the transverse trap electrodes.

The ion trap construction phase also allows learning from existing skills of the BNL ion source and RF groups, plus other groups who have built similar equipment.

Figure 3 shows a simulation of a Coulomb crystal compared to experimental results from the S-POD trap [3]. There are several diagnostics available for ion traps, the simplest being extraction onto a screen (multi-channel plate amplifier and phosphor screen, or pixel detector) to get a two-dimensional projection of the ion distribution. During laser cooling, the ions will emit photons at the rate they absorb them, and this is how the fluorescence image in Figure 3 was produced. A large-angle lens and camera are required to magnify and collect the light efficiently. Further diagnostics include probing the fluorescence signal strength as a function of frequency shift of the laser, which gives a slightly smeared version of the ion velocity distribution in the laser beam axis (this was used in [17]).

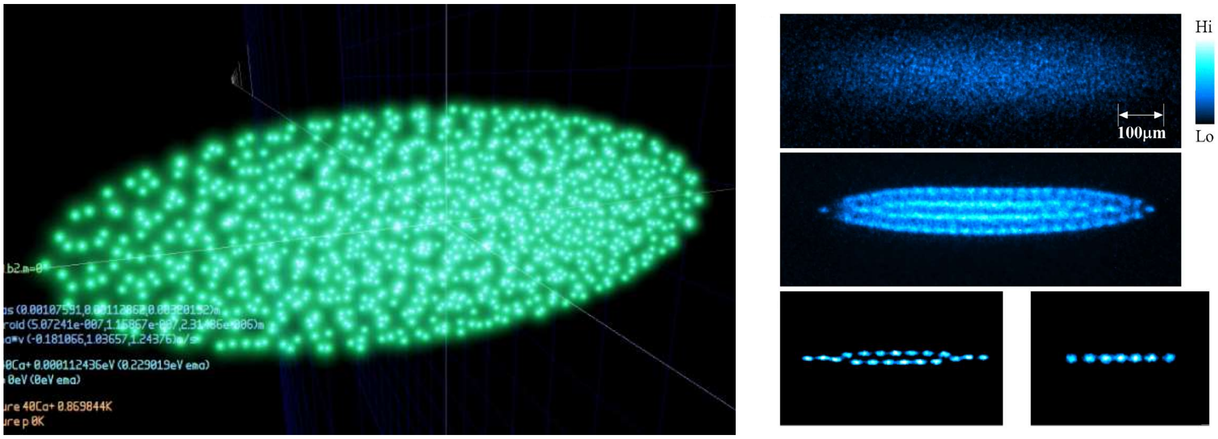

Figure 3. (left) Space charge simulation of 1000 Ca^+ ions in an electrostatic trap. (right) Experimental results from fluorescence diagnostic of an ion trap at the University of Hiroshima [3].

The second core piece of apparatus for this experiment is a beamline of configurable multipole lenses (Figures 4 and 5), which will likely be electrostatic for initial experiments with low energy ions.

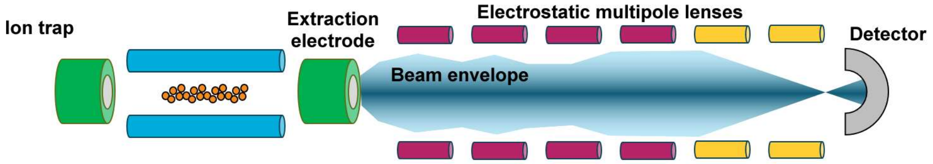

Figure 4. Schematic of beamline; configurable electrostatic multipoles are shown in purple.

While the final few lenses of the beamline will bring the ultra-low emittance bunch to an approximate focus, at the level of precision required here optical aberrations from errors and such effects as spherical aberration will have to be corrected. This is why many more configurable electrostatic multipoles are provided upstream to fine-tune the higher order shape of the ion bunch velocity distribution. These lenses (shown in Figure 5) have 12 independent voltage plates, providing acceleration/deceleration and lens multipole components up to a decapole of both normal and skew type. Initial simulations have shown that optimizing the fields of such lenses can greatly reduce the focal size, canceling each order of optical aberration in turn (see section 3.2). An energy chirp, which can be acquired as the longitudinal electrode is turned off during extraction, or from other time-dependent electrodes, is necessary to also focus the longitudinal axis.

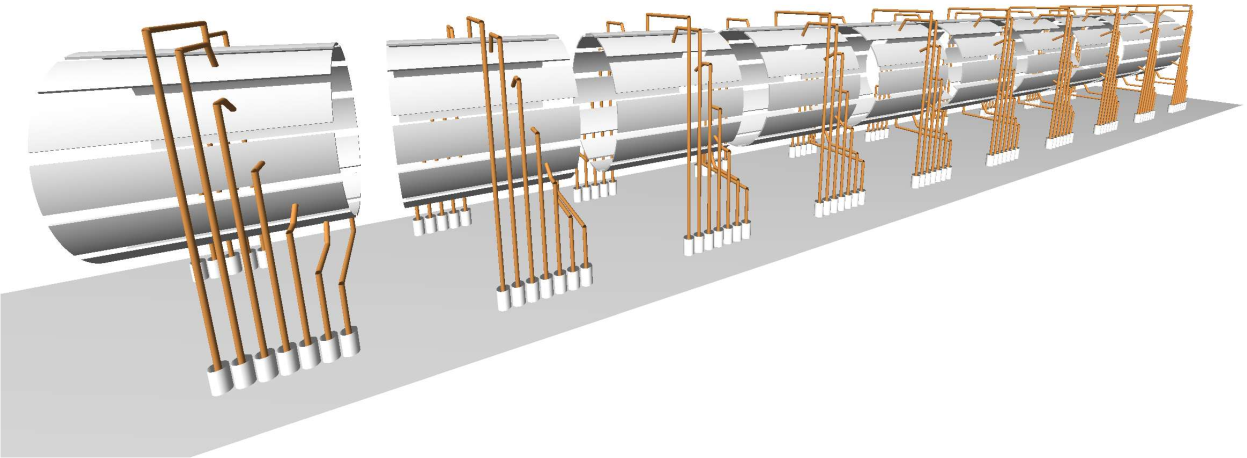

Figure 5. Geometry of beamline containing 10 fully configurable 12-zone electrostatic lenses, with voltage feedthroughs. Mechanical support structure not shown for clarity.

A final detector will occupy the relevant outgoing angles of the focal point, likely a CCD image sensor that can detect individual ions with a digital readout. This will also be useful in verifying that the optics of the configurable beamline agree with model predictions as the lenses are varied, by observing the changing size and shape of the ion bunch. Once the focus approaches the Coulomb limit (most easily attained at low energy and large size initially), scattering of the ions will be observed. This can be analyzed in quite complex ways, which will form a significant thread of mathematical work in this project, but a simple heuristic is that the nearer the ions are to direct collision, the larger the sensitivity of the outgoing angles to the ingoing positions. Slightly changing an upstream dipole lens between two otherwise identical bunch extractions will allow this sensitivity to be measured. More generally, a lot of additional information can be obtained by parameter scans, for example, mapping the sensitivity as a function of other parameter(s) to automatically optimize the focus.

More accurate operation at successively smaller focal sizes may require vibration isolation or feedback in various ways, and the roughly ~$500k total hardware budget contains enough to make some attempts at this. Analysis of the ion scattering pattern to determine which sort of optical aberration needs to be corrected next will also be attempted. Ideas for this include measuring the higher-order moments of the flow distribution of the ions (as is done for analyzing nucleons in RHIC), analytic classical dynamics methods, or even a machine-learning-based approach to predict good optics changes from detector readings.

3.1 Ion Trap Initial Simulation Results

Some simulations of Coulomb crystal formation in a laser-Doppler-cooled ion trap have already been made for this project, to understand the initial particle distribution of the ultra-low emittance bunch. Statistics from an example simulation are shown in Figure 6, in which a group of 500 trapped calcium ions, initially at >20 K temperature, are cooled to ~2 millikelvins. This is comparable to the Doppler cooling limit temperature, which in one dimension is $T_D=\hbar\Gamma/2k_B=0.552$ mK, giving a temperature of 1.66 mK for the three-dimensional case. The reason for the ~30% higher temperature observed in the simulation is suspected to be RF heating of the particles that are not exactly on the trap axis. There are also two phases of the cooling process that can be seen from the graph: one from t=0 to 5 ms and one from 5 to 14.5 ms, before a steady-state Coulomb crystal is achieved. In the first of these, the whole cloud of ions is cooling. In the second, most of the ions have formed a solid crystal at the center of the trap but a few remain in "orbits" that do not hit the crystal. These final ions must wait to be cooled by the laser before the entire distribution is crystallized.

This simulation includes three forces: the electrostatic and RF field of the trap electrodes, the Coulomb repulsion of the ions, and the effect of laser photon absorption and emission. The trap potential, near the center (0,0,0), is approximately

$$V(x,y,z,t) = k_T \sin(2\pi ft)\,(x^2-y^2) + k_L\,(z^2-\tfrac{1}{2}x^2-\tfrac{1}{2}y^2),$$

where, in this simulation, k_T=2000 V/cm^2, k_L=0.111 V/cm^2, and f=10 MHz. The forces from this potential are integrated with the 4th order Runge-Kutta method with timestep 1/200 of the RF period (0.5 ns here), which is small enough to give good energy conservation.

The simulation can be run on the GPU (still at double precision) and the Coulomb force is implemented by an all-to-all pairwise repulsion step that happens between the Runge-Kutta steps and applies a velocity kick, which still conserves phase space volume.

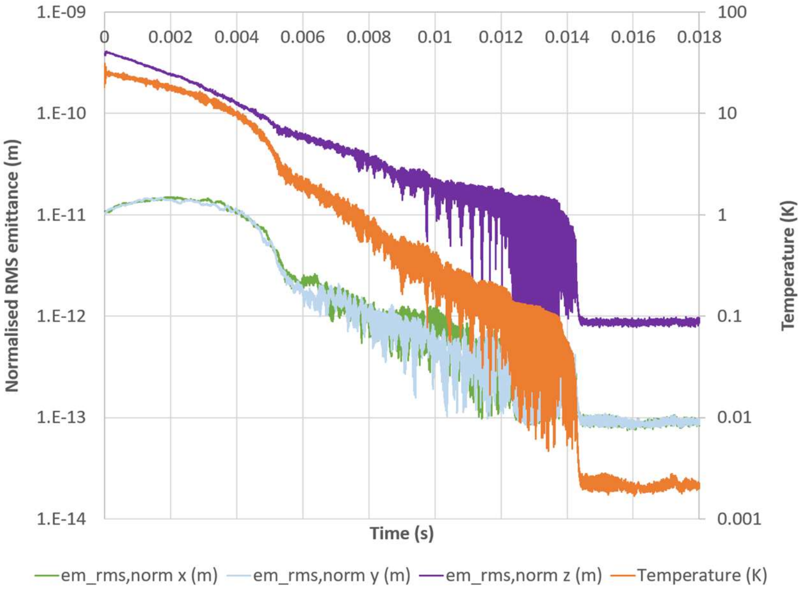

Figure 6. Laser Doppler cooling simulation of 500 $^{40}Ca^+$ ions forming a Coulomb crystal in a trap. Final emittances are just under 10^{-12} m longitudinal and 10^{-13} m transverse, with a final temperature of 2.14 mK.

The average force from the Doppler cooling laser is difficult to model because it only applies in a small velocity band of a few m/s around the velocity where ions are at the center of the absorption linewidth. For example, the 397 nm laser for calcium has a frequency of 755 THz and the 23 MHz linewidth is a relative fraction 3.05×10^{-8} of that, or 9.13 m/s when multiplied by c.

Ions in the simulation can move through this entire cooling-relevant velocity band in a fraction of a single timestep. The solution adopted here is to model the absorption and emission of individual photons, because this happens roughly at the speed dictated by the linewidth (e.g., 23 MHz), while the timesteps here are much faster (2 GHz). Random points during a timestep are selected using a Poisson process and the kicks for absorption of a photon applied depending on the velocity at that time. The rates are chosen to be consistent with those derived from the quantum system of a two-level atom in [14].

An important question in this research is whether the ultra-low emittance of the Coulomb crystal can be preserved during extraction from an ion trap. A worst case can be modeled by unmatched "uncontrolled" extraction, where one of the longitudinal confinement electrodes is simply switched off and the bunch leaves the trap in that direction. To model this, the electrostatic potential was re-implemented as coming from six point sources 3 cm from the trap center in the ±X,Y,Z directions, which allows one of them to be set to zero and for the bunch to experience a consistent potential outside the trap that has realistic behavior at large distances. The results of such a simulation are shown in Figure 7, which shows that although there is emittance growth of about 10× longitudinally and 4× transversely, this does not undo the multiple orders of magnitude reduction from cooling. Adding further electrodes or lenses could improve the match of the bunch shape to the potential in the external channel, which in similar accelerator settings translates into better preservation of emittance.

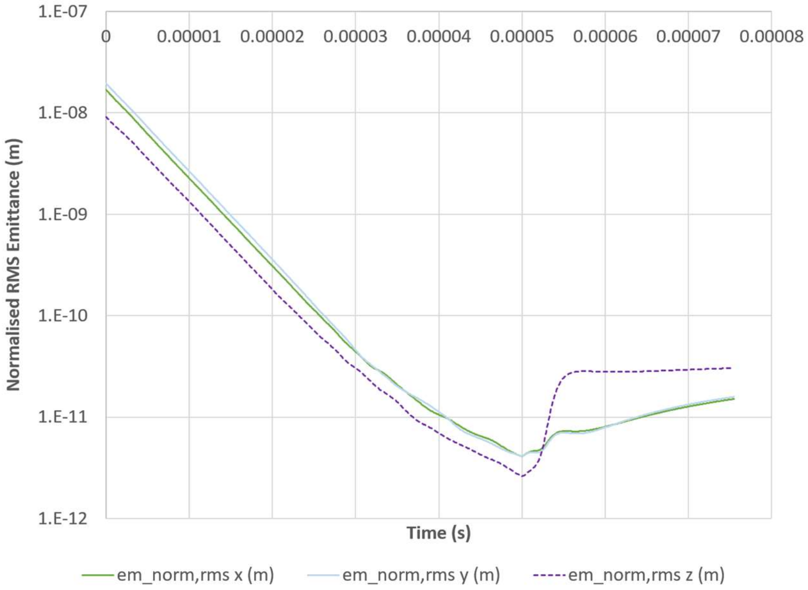

Figure 7. Simulation of unmatched extraction of a Coulomb crystal from an ion trap, showing emittance evolution. Cooling rate has been artificially speeded up in the first part of the simulation to form the crystal.

3.2 Focusing Beamline Initial Simulation Results

A bunch containing N=500 $^{40}Ca^{+}$ ions with the parameters from section 1.5 has been simulated traveling through a beamline of configurable electrostatic lenses, which were optimized to minimize RMS bunch size 15 μs later. Results are shown in Figure 8 (left) as a function of iterations of the optimization algorithm. The progress of reducing the focal size can be seen to happen in progressively more difficult jumps, which take more iterations to find, with each further reducing the focal size by roughly an order of magnitude.

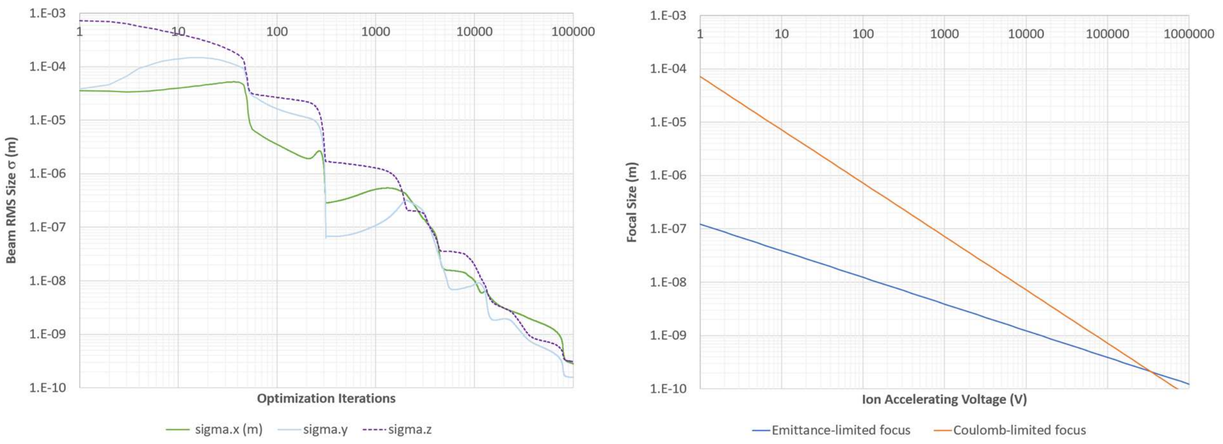

Figure 8. (left) Progressive optimization of focal size for a 500 Ca$^+$ ion bunch in a simulation of a beamline of configurable electrostatic lenses. (right) Theoretical dependence of focal size on acceleration voltage.

The electrostatic beamline simulated here consists of eight circular rings of 16 independently-powered electrodes each, making a total of 128 optimization variables. The rings have a radius of 8 cm and are spaced 12.5 cm apart longitudinally. The input bunch for the optical simulation was pre-accelerated to an energy of 1 keV with an energy chirp of ±5 eV along its length. A feature of the 3D focusing produced from the optimization is that the bunch is sent off-axis and around bends to couple the energy spread with the path length to enable longitudinal bunch compression.

It should be noted that this simulation does not have space charge and assumes a zero temperature (zero emittance) initial beam rather than the small Doppler limit temperature of ~2 mK. This is because the simulation is intended to demonstrate that multiple orders of purely optical aberration can be corrected. The real focal size limits from theory are shown on the right-hand side of Figure 8 and depend on beam energy.

Appropriateness of the Proposed Method or Approach

The first part of this project is based on laser-cooled ion traps that exist elsewhere, so it is achievable. If this part of the experimental program becomes stuck on some issue, there are several groups around the world who already have laser-cooled ion traps and may be able to offer advice.

The addition of a beamline to the ion trap is novel but the beamline itself is constructed from very normal components such as electrostatic lenses that are widely used in low-energy beamlines. However, the scientific output is large because many experiments that involve focusing the ultra-low emittance bunch become possible.

Potential problems at the "join" between the two sections, such as extraction of the bunch from the trap, are limited by the fact that many existing facilities use extraction from the trap as a way of getting the ions to a diagnostic screen. A simulation has also been run to see if the ultra-low emittance of the bunch dramatically degenerates after extraction and the effect is not too bad.

Optics optimization simulations are in progress for higher-order correction at the focus, with encouraging initial results reported in section 3.2. The simulated focused bunch takes on curved shapes as it gets smaller, showing higher-order effects are present, but still shrinks to below the limits expected from Coulomb repulsion. The required energy chirp at the start of the beamline can naturally be obtained from the time-varying voltage of the extraction (or other) electrode. Formation of the focus only requires the (x,y,z) final positions to be constrained and not the angles, and only applies to the effectively three-dimensional subset of six-dimensional phase space that the ultra-cold beam occupies (by its initially very low velocity spread). It is therefore not anticipated that conserved quantities or symplecticity of the dynamics will prevent formation of the focus to any desired degree of accuracy.

It is definitely possible that vibrations and experimental jitter will have a noticeable effect on the focus as its size becomes less than a micron. As mentioned earlier, there is enough budget to add feedbacks, and these could be of several types. Vibrations can be measured by interferometer or accelerometer, or by looking at the effects on the ion scattering at the focus (although that can only be sampled at a limited rate). The mitigations can include suspension isolation or feedbacks from piezo movers. Electrical noise can be mitigated in various ways, such as shielding, adding capacitors, or (somewhat extremely) by disconnecting the supplies from the electrodes, leaving a static charge without ripple.

As mentioned earlier, several ways of analyzing the scattered ion output distribution are possible to detect the size of the focus, including a fairly simple "sensitivity" rule where a smaller focus is more sensitive to initial conditions. This configurable beamline setup with a screen is very amenable to parameter scans for benchmarking the basic optics and beam distribution before attempting more complex corrections.

Practically, the large number of channels and electrical vacuum feedthroughs for controlling the configurable beamline might be a cause of concern. However, modern electronics can produce multiple channels easily and vacuum feedthroughs are available for multi-pin connectors. Careful choices of the accelerating and focusing lens voltage scales will make this easier; for example, limiting ~10kV to a platform containing the ion trap (for acceleration) and then using multiple channels of smaller voltages ~100V on the configurable beamline.

Overall, the addition of a low-energy beamline for manipulating an extracted ultra-low emittance bunch is a very logical next step for testing the applications of laser-cooled ion traps for accelerators that require a bunch in motion.

4.0 Timetable of Activities

This proposal funds part of the Principal Investigator's salary, a student, and $540k of hardware over five years. The rough schedule is as follows.

- **Year 1: Produce, trap, and measure ions.** This phase involves procurement of the vacuum chamber and attaching the gas source or oven and an electron beam to ionize the atoms. A basic linear Paul trap and diagnostic (perhaps as simple as a Faraday cup) will be placed in the vacuum chamber. The goal at this point is to see an ion signal that is correlated with the drop of the trap extraction electrode, to show that ions were trapped and then released. Better diagnostics such as screens can then be tested to observe the ion transverse distribution.
- **Year 2: Add laser and cool ions.** In this year, the major purchase is a tunable diode laser with reference optical cavity and wavemeter so that it may get into the narrow frequency range to generate fluorescence of the trapped ions. A lens and camera are also needed to capture images of this fluorescence and the ion distribution. Observing the size of the extracting ion beam after being exposed to the laser for different amounts of time should give evidence of cooling.
- **Year 3: Extract ions from the trap into beamline chamber.** A second vacuum chamber will be joined to the ion trap chamber, with an insulating column so that the potential of the trap can be raised, generating acceleration. The second chamber will also have screen and Faraday cup diagnostics to characterize the distribution of the cooled bunch once it has been extracted from the trap. A basic electrostatic beamline can be added to the second chamber this year.
- **Years 4 and 5: Accelerate and focus ions, optimize the focus.** At this point, most of the experimental parameters, such as accelerating voltage, beamline lens settings and ion trap and cooling, should be controllable. This leaves the final two years for an experimental program of trying to focus the ultra-low emittance bunch and studying the minimum achievable focal size as a function of energy.

Software, simulations, and control systems will continue to be developed throughout the entire period.

The student's involvement will depend on their interests, whether experimental, simulation, or theory. The configurable beamline offers many possible practical experiments or situations to simulate or analyze.

Competency of Personnel and Adequacy of Proposed Resources

Brookhaven National Laboratory is an ideal environment for developing an ion source and beamline, as it has already developed many ion sources such as EBIS [19] and is a national accelerator lab containing the USA's largest operating collider. The experimental environment and support groups (vacuum, survey, fabrication, safety, etc.) are excellent and are experienced in relevant situations relating to accelerators.

The PI Stephen Brooks is an experienced accelerator physicist who has worked at BNL for nine years, including the design, simulation, construction, and operation of accelerators and magnets.

The proposal budget must pay 50% of the PI salary, with overheads, as well as a student (see Appendices 5 and 6), and $540k over the 5 years for hardware purchases. The laser system is one of the larger items and costs $110k, with similar amounts for the two experimental chambers. The experiment does not require unusually high voltages, energies, or bandwidths and is not physically large. Some of the feedback and controls hardware (piezo movers, optical interferometers) can be made from mass-produced components.

Principal Investigator Leadership Capabilities

Stephen Brooks has a research interest in novel accelerator designs and testing thus-far-unimplemented ideas. He ran the first computer tracking simulation of the VFFA [20], a fixed-field accelerator that has vertical orbit excursion with energy, as opposed to the radial orbit excursion of a cyclotron or FFA. This idea had been proposed in the 1950s and 60s, e.g. [21], but research had been dormant until this point.

FFA accelerator designs led to an interest in magnet technology, leading Brooks to develop a method to increase the field accuracy of permanent magnets [22] for an energy-recovery linac (ERL) non-scaling FFA demonstrator called CBETA [23], built at Cornell University. This machine was the highest energy (150MeV) non-scaling FFA machine yet run and the optics in the permanent magnet lines performed well. Brooks oversaw the magnet procurement, production, and field tuning for CBETA in 2018, making over 200 permanent magnets, and took part in machine commissioning shifts in 2019-20. During the final shifts, eight turn (four accelerating, four decelerating) ERL operation was achieved at CBETA.

Brooks received a Lab-Directed R&D (LDRD) award on the topic of "High-Gradient Permanent Magnets for Emerging Accelerator Applications," providing $400k over the period of Oct 2021-Oct 2023. This research is a logical continuation of the CBETA permanent magnet designs to higher field: 1.53 Tesla peak field in the beam region rather than 0.587 T, while maintaining accuracy. He built a magnet with this spec including an open midplane for compatibility with the CEBAF 20 GeV energy upgrade design study.

Recently, Brooks has organized a monthly "Ion Traps for Accelerators" video meeting, with participation from BNL, Stony Brook University, and Rutherford Appleton Laboratory (RAL) in the UK.

Mathematical ideas and optimization are often used in Brooks' designs. His thesis designed a pion capture beamline for the proposed neutrino factory [24] using an evolutionary optimization algorithm deployed across a distributed computing project with hundreds of volunteer users. The permanent magnet designs for CBETA and LDRD were developed using many-variable optimizers to determine their geometry.

Brooks has been invited to give talks twice at the IPAC worldwide accelerator conference: once on the development of VFFAs and related cyclotron variants [25], and once on the theoretical limits of particle acceleration and focusing at high energy [26]. The second of these partly inspires the ultra-low emittance focus study in this proposal.

5.0 Summary

In this project, a laser-cooled ion trap will be built that produces ultra-cold bunches of ions with an emittance a million times smaller than in traditional accelerators. These will be extracted and, for the first time ever, tightly focused in a beamline to prove the minimum focal sizes attainable as a function of energy. If implemented in a future multi-GeV facility, these extremely small foci enable energy efficient colliders and production of hyper-dense matter, including exotic configurations of nuclear matter.

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