MODIFIED HALBACH MAGNETS FOR EMERGING ACCELERATOR APPLICATIONS*

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Abstract

The original circular Halbach magnet design creates a strong pure multipole field from permanent magnet pieces without intervening iron. This design has been extended recently at the CBETA 4-turn ERL [1, 2], whose return loop includes combined-function (dipole+quadrupole) Halbach-derived magnets, plus a modular system of tuning shims [3] to improve all 216 magnets' relative field accuracy to better than 10^{-3}. This paper describes further modifications of the Halbach design enable a larger range of accelerator applications in the future: (1) open-midplane designs to allow synchrotron radiation in light sources and other high-energy electron rings, ERLs or RLAs to escape. (2) Quadrupole magnets with an oval aperture allow larger gradients than a circular aperture, provided the beam is more extended in one axis than the other, as usual for a quadrupole in a focussing system. These can be used in compact hadron therapy gantries. (3) New collider complexes often require multiple rings for acceleration or top-up, accumulation, collision and cooling. Multi-aperture permanent magnets are possible to cheaply and compactly build ring systems with several stable orbits separated by a few cm.

INTRODUCTION

Halbach type permanent magnets (PMs) or Halbach arrays have some established uses in accelerators: most commonly as wigglers or undulators in light sources, but also appearing as quadrupoles inside the drift tubes of linacs to make best use of limited space. The CESR collider at Cornell installed SmCo Halbach quadrupoles near its interaction region [4] to improve focussing without having to use superconducting magnets. In a more extensive beam dynamics use, hybrid permanent magnets containing iron poles were used for the main bending field of the Fermilab recycler [5].

The CBETA ERL [1, 2] used a beamline of 216 Halbach permanent magnets to return four different electron energies (42–150 MeV) to its linac. This was the first use of 'pure' permanent magnets without iron for the main bending field of an accelerator facility and enabled a compact lattice cell via the elimination of flux cross-talk between the iron poles of adjacent magnets. The CBETA magnets also included a combined-function magnet design and shimming method to achieve the required fields, as detailed in [3].

Several further uses of Halbach-type permanent magnets have been proposed, a (non-exhaustive) selection of which are shown in Table 1. They are roughly sorted in increasing order of required gradient, which correlates with decreasing physical aperture.

OPEN-MIDPLANE MAGNETS

A common requirement seen in Table 1 for high-energy electron facilities is to have the horizontal midplane of the magnet free of permanent magnet material. This is because the powerful beams of synchrotron radiation produced by multi-GeV electrons in a strong magnetic field would irradiate the material and potentially demagnetise it.

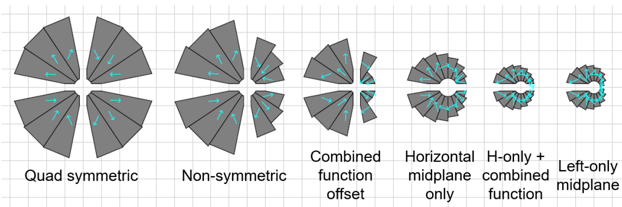

Figure 1: Six ways of constructing an open-midplane Halbach magnet with the same field strength and quality.

Several ways of achieving an open-midplane modified Halbach design are shown in Fig. 1. A four-way symmetric design intrinsically cancels all harmonic errors except 12, 20, 28...-pole but requires a material-free midplane in the vertical axis that is not strictly needed. Breaking some of the symmetry and re-optimising for minimal harmonic errors gives several designs that use less material for the same achieved field and field quality. The requirements for this benchmark are given in Table 2.

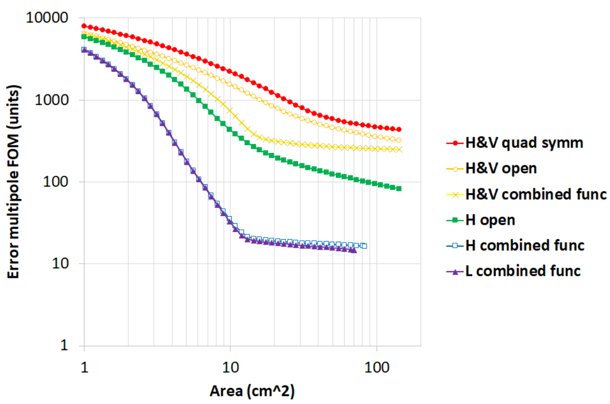

Figure 2: Magnet cross-sectional area vs. field quality for six types of open-midplane magnet.

If a 'unit' of field is defined as 10^{-4} of the strength of the main pole (quadrupole here) at the good field radius, then

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Table 1: Magnet parameters for main repeating cells of: the recent hadron therapy gantry design from [6, 7], the "Complex Bend II" NSLS-II upgrade light source lattice [8] and the CEBAF 20 GeV upgrade working design using a fixed-field multi-energy arc [9]. Magnets for the PETRA-IV light source and ILC damping ring are also shown, but marked $^{(o)}$ to indicate permanent magnets are an option under study, not the baseline proposal.

Project	Magnet name	Dipole at $x = 0$ (T) (Max beam field)	Gradient (T/m)	Aperture radius (mm)	Open midplane?
CBETA [1–3]	QF	0 (0.2891)	-11.5624	46.2	No
(already built)	BD	-0.3081 (0.5868)	11.1475	43.2	No
ILC damping ring$^{(o)}$	quads	0	15	32.5	Yes
PETRA-IV$^{(o)}$	combined func.	0.1976, 0.2861	25.83, 38.94	12.5	Yes
	quads	0	50 to 95	12.5	Yes
Hadron therapy gantry	QF	0 (1.364)	155	13.8	No (protons, but
(Trbojevic design)	BD	1.8 (2.527)	-97	12.5	oval aperture)
NSLS-II upgrade	F	0.26	250	5	Yes
(CBII lattice)	D	0.49	-250	5	Yes
CEBAF 20GeV	F	0.8827 (1.910)	321.05	5	Yes
upgrade	D	0.8827 (1.483)	-187.47	5	Yes
Plasma accelerators [10]	quads	0	100 to 500	3 to 12.5	No

Table 2: Open midplane magnet benchmark parameters.

Parameter	Value	Unit
Dipole at good field centre	0.49	T
Gradient	-250	T/m
Good field radius	2	mm
Aperture radius	5	mm
Midplane slot full height	4	mm
Remnant field B_r	1.12	T

an error figure of merit may be defined as

$$\text{FOM} = \sqrt{\sum_n (b_n - b_n^{\text{goal}})^2 + (a_n - a_n^{\text{goal}})^2},$$

where b_n and a_n are the normal and skew integrated field harmonics in units. This is also proportional to the RMS field error on the circle at the good field radius.

Figure 2 shows how the field error FOM changes as a function of PM material area allowed to be used in the geometry optimisation. The "H&V" designs have horizontal and vertical open midplanes, the first retains quadrupole symmetry and the third recenters the magnet good field region to be implemented as a combined-function magnet rather than a displaced quadrupole. The "H" designs have only horizontal open midplanes, the first is a displaced quadrupole and the second is combined-function. The final "L" design only has an open midplane on the left-hand side (towards the outside of the ring), which makes little improvement in this case because the newly filled right-hand side of the magnet is the low field side. The "L" variant makes more improvement for a +250 T/m magnet with the same sign of dipole.

OVAL APERTURE DESIGNS

Beams in accelerators are often not circular, either due to dispersion and large momentum spread, or simply that the maximum beta function in one plane typically occurs in the focussing magnet for that plane, while the other beta function is smaller. This section investigates whether PMs with non-circular apertures offer any improvement.

PMs with oval apertures were proposed by Halbach himself [11] as an analytic design with inner and outer edges elliptical. Combining a circular aperture with an elliptical outer edge has been benchmarked in [12]. There are analogous designs for superconducting magnets because PMs act similarly to pure current sources. A *combined-function*, oval aperture superconducting magnet is described in [13].

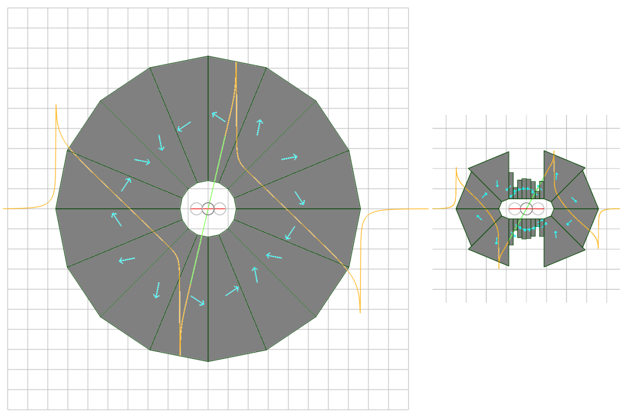

Figure 3: A 155 T/m quadrupole designed with a circular aperture and an oval aperture (cm grid shown).

High-performance PMs are required for the hadron therapy gantry proposed in [6, 7], which uses the non-scaling fixed-field accelerator (NS-FFA) principle [14] to transmit proton energies from 65–250 MeV simultaneously. Figure 3 shows the main focussing magnet for this lattice implemented as both a circular and an oval aperture Halbach quadrupole. It requires a 155 T/m gradient and 27.6 mm horizontal aperture but the oval magnet's vertical aperture is reduced to 10 mm. This aperture is formed from a rectangle and two R5 mm semicircles to provide a uniform 5 mm beam clearance, rather than being an actual ellipse. The original magnet uses 170.9 cm^2 of PM material but the oval aperture version uses 24.0 cm^2, a seven-fold decrease. Both simulations achieve 10^{-4} relative field accuracy on the midplane.

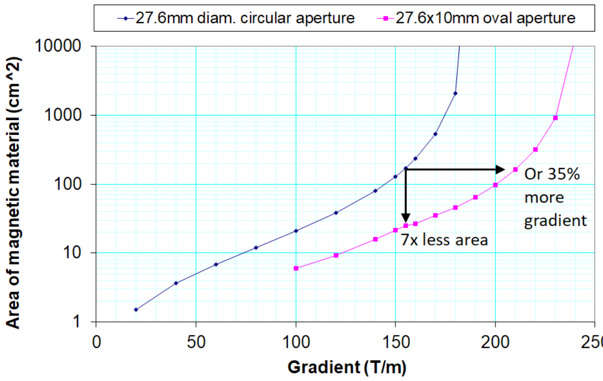

Figure 4: Performance comparison of oval aperture to circular aperture Halbach quadrupoles.

Figure 4 shows how the area of these designs changes as a function of required gradient, where Halbach quadrupoles always have a hard limit on their strength when the area tends to infinity. A strong NdFeB grade with $B_r = 1.4$ T was used in this study but fields scale linearly with B_r.

MULTIPLE APERTURE DESIGNS

Two-aperture superconducting magnets have been used in colliders such as the LHC but similar designs at lower fields are possible using permanent magnets. Multiple apertures may be economic to transfer several beams around the same tunnel arc on a single beamline girder, or even in a unified vaccum chamber. As well as counterrotating beams, modern colliders can require accumulation, bunch merging and cooling functions that are optimal at different bunch lengths than collisions. Additional apertures can perform these functions simultaneously with collisions and serve as a reservoir of new beam to inject in a single turn, which increases average luminosity by avoiding halts to accumulate or cool beams.

Figure 5 shows a two-aperture Halbach magnet designed using the parameters in Table 3. These parameters were originally part of the ~20 GeV electron side of a permanent magnet electron-ion collider and its focussing and defocussing magnets have been joined in a single magnet.

The two-aperture magnet has an area of 15.2 cm^2 of PM material, while separate magnets with the same parameters would have areas of 11.3 cm^2 (left) and 15.4 cm^2 (right), summing to 26.6 cm^2. The design benefits from the field (graphed in Fig. 5 in orange) forming a 'pyramid' shape

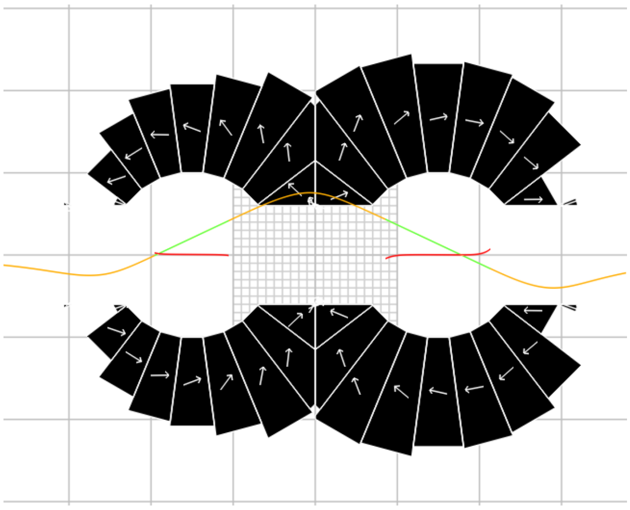

Figure 5: A Halbach magnet with two apertures.

Table 3: Two-aperture magnet benchmark parameters.

Parameter	Left	Right	Unit
Dipole at centre	0.2190	0.1382	T
Gradient	49.515	-49.515	T/m
Good field radius	4.5	6.4	mm
Field error FOM	4.7	18.0	units
Parameter		**Value**	**Unit**
Aperture centre separation		30	mm
Aperture radii		10	mm
Midplane slot full height		12	mm
Remnant field B_r		1.15791	T

between the magnets. If the left and right are swapped, the resulting area of a two-aperture magnet would be 28.9 cm^2.

UPCOMING PM R&D AT BNL

Two lab-directed R&D proposals have recently been made at BNL. One is an exploratory proposal ($400k in FY22,23) to do a parameter scan of high-field modified Halbach magnets as described in this paper. As well as field quality tests, adaptation of the CBETA tuning rod process to smaller geometries will be investigated. The smaller apertures may require the 3D printed plastic rod holder (~3.1 mm thick) to be replaced by a tape with thin iron rods adhered, or the rods to be placed in grooves integrated into the vacuum chamber, if this is inside the magnet. A radiation exposure test on the RHIC beam dump may be possible, as was done once before in 2015. Field loss from demagnetisation in various PM materials will be a key limit to define.

The second proposal under development is to produce a full-field, full-gradient, full-length engineering prototype of the main magnet section for the NSLS-II upgrade [8]. This would be a project-specific proposal of >$1M. These 'complex bend' lattices can reduce equilibrium horizontal emittances in a 3 GeV light source from 2.1 nm to 65 pm [15].

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