PROPOSED ULTRALOW-EMITTANCE BEAM SOURCE FOR HIGH-LUMINOSITY HADRON COLLIDERS*

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Abstract

Laser Doppler cooling of ion bunches in a Paul trap is a demonstrated method of achieving millikelvin bunch temperatures, with the ions forming a Coulomb crystal with a solid-like structure. This is proposed as a source for accelerators that would be a factor 10^5 lower in emittance than conventional plasma sources. Methods to transport the crystalline bunch while limiting emittance growth are examined, including a novel ring in which the bunch maintains a fixed orientation relative to the outside world (i.e. does not rotate with the ring as usual). In this geometry, magnetic focussing can confine all three dimensions of the bunch without RF. This ring can circulate a 3D crystalline bunch with heating rates of less than 1 K/s.

LASER COOLED ION TRAPS

Laser Doppler cooling is an established method for studying atoms and ions at low temperatures [1]. Several trap geometries can be used with this method but the linear Paul ion trap, with DC electrostatic confinement longitudinally and ~MHz range RF quadrupole transversely, has been used for accelerator applications. The SPOD experiment [2] at Hiroshima University has prepared Ca^+ Coulomb crystals at rest, where the individual ions have vibration velocities of a few m/s and can be seen in a fixed lattice via their fluorescence light during cooling [3]. A Paul trap without cooling has also been built for accelerator physics reasons in the IBEX experiment [4] at Rutherford Appleton Laboratory, where the resonances of the ion cloud under space charge forces can be studied.

Cooling Simulation

A simulation code that runs on the GPU using OpenCL has been written to simulate cooled ion traps. 4^{th} order Runge–Kutta steps in the trap potential are interleaved with pairwise space charge kicks. The ions have a ground and an excited state, the transitions between them governed by a statistical model whose rates agree with the quantum description of a two-level atom in a laser beam. These excitations and de-excitations come with appropriate $\Delta p = \hbar k$ momentum kicks, where it is important to apply the events at truly random points within each timestep, so the short periods where the Doppler process is active are not jumped over.

The results of such a simulation are shown in Figure 1, where the eventual configuration of ions was a long Coulomb crystal several ions wide. Two phases of cooling can be seen: the first occurs before formation of a solid Coulomb crystal

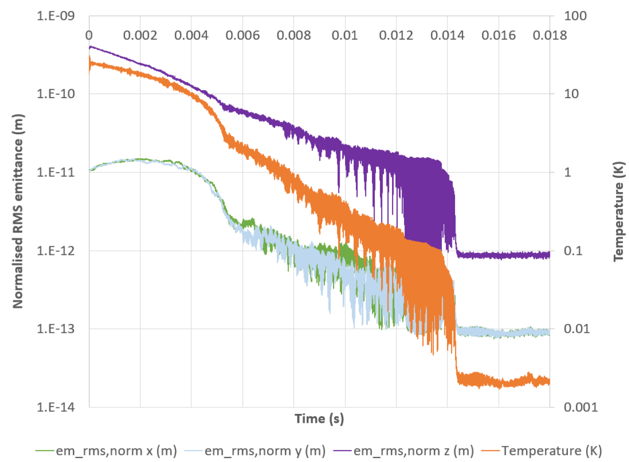

Figure 1: Laser Doppler cooling simulation with $N = 500$ ions.

core from the initially gas-like ions, while the second ends when the small fraction of remaining ions 'orbiting' the solid core eventually lose enough momentum to merge with it. The final temperature is a small multiple of the predicted Doppler limit ($T_D = 0.552$ mK here). The RF nature of the transverse focussing excites some vibrations that take the temperature above the limit.

The final emittances are very small. For a shorter trap producing a spherical bunch, the scaling

$$\frac{\sigma_{x,y,z}}{N^{1/3}} \simeq 25\,\mu\text{m},$$

$$\frac{\epsilon_{x,y}}{N^{1/3}} \simeq 7 \times 10^{-14}\,\text{m}, \qquad \frac{\epsilon_z}{N^{1/3}} \simeq 4 \times 10^{-14}\,\text{m}$$

was observed in simulations from $N = 100$ to 81920 ions, where ϵ denotes normalised RMS emittance. The crystal temperature was consistently ~3 mK. This emittance is related to temperature via $\epsilon \simeq \sigma_x \frac{\sigma_v}{c} = \frac{\sigma_x}{c}\sqrt{\frac{k_B T}{m}}$.

Increasing Throughput

The maximum possible ion population in existing traps is around 10^7 [2, 5]. With the cooling time of up to 20 ms, a 50 Hz repetition rate source would give an average current of only 80 pA. However, existing sources are not optimised for throughput. The trapping electrode voltages are typically 150 V or lower, which could be increased by two orders of magnitude with a high voltage setup, allowing 10^9 ions to be trapped. A more drastic improvement would be possible using a linear cooling channel, where multiple bunches move along at ~10 km/s so that by the end of a ~200 m-long (possibly coiled) channel they would have completed cooling. This channel would resemble an RFQ but operate at

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lower frequencies and velocities; it would also be immersed in the cooling laser light, which is not particularly intense (48 mW/cm^2) and not completely absorbed by the ions, so could be reused multiple times with mirrors. Maximum repetition rates would be of the order 1 MHz, corresponding to ~1 cm bunch spacing in the channel and producing an average current of 160 μA of ultra-low emittance beam.

Related Studies

Extraction from the trap through a non-linear field has been simulated in [6], where relatively simple changes to the extraction system can limit 6D emittance growth below a factor of two. Creation of a high density focal point from an ultra-low emittance bunch, correcting for optical aberrations, was studied in [7] with computer optimisation.

LUMINOSITY ESTIMATE

Parameters of an ion trap bunch are compared to those of a typical plasma ion source in Table 1.

Table 1: Simplified Luminosity Estimate

Parameter	Ion source	Ion trap
N	10^9	10^5
$\epsilon_{\text{norm,rms}}$	10^{-7} m	3.2×10^{-12} m
Initial condition:		
σ_x	1 mm	1.2 mm
σ_v	30 km/s	0.8 m/s
T	4.3 MK	3.2 mK
At focus:		
σ_x^*	9.4 nm	0.3 pm
σ_θ^*	10 mrad	10 mrad
L/bunch	9×10^{28} cm^{-2}	8.8×10^{29} cm^{-2}

The ion trap bunch can be focussed far smaller than the conventional bunch, albeit with a lower population of particles. Luminosity per bunch calculated via $L = \frac{N_1 N_2 f N_b}{4\pi\sigma_x\sigma_y} \Rightarrow L/\text{bunch} = \frac{N_1 N_2}{4\pi\sigma_x\sigma_y}$ shows that the low emittance bunch produces almost ten times the luminosity despite having 10^4 times fewer ions. This increase in specific luminosity would enable highly energy efficient linear or ERL colliders, with lower beam currents giving reduced power consumption and collective effects. The picometre-level focal point does not require stronger focussing magnets because σ_θ^* is the same as the conventional case. Stabilising against vibrations would be the main challenge, while optical aberration correction and longitudinal compression for a point-like focus are discussed in [7].

From the total Ca-Ca cross section $\sigma_{tot} = 0.9\times10^{-24}$ cm^2, a straightforward estimate would predict 8×10^5 events from colliding the ion trap bunches. This exceeds the bunch population of 10^5, so the interaction point operates in the opaque bunch regime where the entire beam is consumed in one pass. This means a collider ring is not needed, although rings may be used to reduce the cost of acceleration relative to linacs. Taking instead the saturated luminosity per bunch crossing of $2N/\sigma_{tot} = 2.1 \times 10^{29}$ cm^{-2}, a 1 MHz bunch repetition rate would give $L = 2.1 \times 10^{35}$ cm^{-2}s^{-1} from 16 nA of beam current.

BEAM TRANSPORT

A solid phase material such as a Coulomb crystal only has one velocity at any given position, whereas a gas has a velocity distribution. Figure 2 shows this difference: the gas is an area in phase space whereas the solid is just a line.

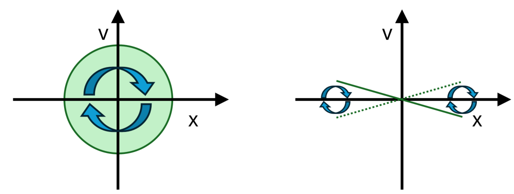

Figure 2: Phase space of a gas phase beam (left) and solid phase beam (right) being transported in a beamline.

The usual phase space rotation in beamlines is not appropriate for a solid, since at 90 degrees phase advance the line would be vertical, corresponding to the particles smashing through each other in that axis. This is desired at the final collision point but not before because it would cause intra-beam scattering. Roughly zero net phase advance is therefore required, by balancing the external focussing forces with space charge repulsion (i.e. operating at the space charge limit). Divergence from the emittance itself is suppressed by the strongly coupled solid structure. Small errors in bunch size cause the dynamics shown on the right of Figure 2, where the bunch 'breathes' around a stable size.

Since the crystal has an internal structure in which forces are balanced, changing the aspect ratio by more than a few percent causes heating (although uniform scaling is allowed). This can be accommodated with a gentle focussing structure, either low phase advance cells or weak focussing in a ring.

STORAGE RING

A storage ring is desired with a low heating rate, so that a Coulomb crystal injected into it will survive as long as possible even without cooling. Okamoto has shown [8] that trying to cool from a gaseous beam to a solid in a ring requires going through a peak IBS heating regime and thus requires powerful cooling. This paper proposes that injecting an already cold crystal will alleviate this problem and reduce cooling requirements in the ring. (Acceleration in such a ring will be studied in future work.)

Field Model

Consider motion of equal-energy particles in a uniform vertical magnetic field. All orbits are equal radius circles and are translated copies of each other, so relative positions of particles in the bunch are constant *in the lab frame* rather than rotating with the direction of travel. If space charge were zero, this motion keeps a crystalline structure stable and avoids rotation, which at relativistic speeds may require large changes in particle momentum.

To this constant field, M small azimuthal oscillations can be added to provide focussing. The midplane field

$$B_z(r,\theta,0) = B_0 + \sum_{j=0}^{n_c-1} c_j r^{M+2j} \sin(M\theta)$$

provides a form of this that has a simple Maxwellian extension off the midplane [9]. The transverse field of the oscillations $b(r) = \sum_{j=0}^{n_c-1} c_j r^{M+2j}$ can be made any shape locally by increasing n_c, the number of c_j values chosen.

As the bunch does not rotate with the ring, focussing at the ring radius R (from $b'(R)$) will first act on the 'North-South' axis of the bunch and then on the 'East-West' axis 90 degrees further around the ring, while the vertical focussing is always active.

Definitions of Bunch Temperature

Temperature is often defined via average kinetic energy as $\frac{3}{2}k_B T = \langle E_k \rangle = \frac{1}{2}m\langle|\mathbf{v}|^2\rangle$ but accelerator bunches may be travelling at high speed, making the naive temperature $\sim 10^{12}$ K, or be changing shape due to the lattice. To measure only 'random' movement of particles and not these collective motions, define

$$T_n = \frac{m}{3k_B}\langle|\mathbf{v} - \mathbf{p}_n(\mathbf{x})|^2\rangle$$

where $\mathbf{p}_n(\mathbf{x})$ is the n^{th} order polynomial least squares fit to $\mathbf{v}(\mathbf{x})$. This means that T_0 removes bunch average velocity. T_1 also removes linear optics motion, giving a quantity similar to the emittance-based definition in [8]. T_2 also removes 2^{nd} order motion, but setting n too high relative to the number of particles will start to 'overfit' and subtract random motion.

Tracking Results

Table 2: Simulation Parameters

Parameter	Value	Unit
Number of ions N	2560	
Ion species	Ca^+	
Bunch size $\sigma_{x,y}, \sigma_z$	481, 265	μm
Kinetic energy per mass	3.257	MeV/u
Main field B_0	1	T
Mean radius R	10.38448	m
Oscillations per turn M	20	
Oscillation terms n_c	1	
Oscillation amplitude $c_0 R^M$	0.081665	T

It was found that the simplest case with $n_c = 1$ and oscillation field $b(r) \propto r^M$ produced stable dynamics for a ring with parameters given in Table 2. The bunch shown on the left of Figure 3 was formed in an ion trap cooling simulation and placed in the ring tracking simulation. After 63.3 ms (24170 turns) it was still solid as shown on the right of the figure. The temperature evolution is plotted in Figure 4 with interesting second-order oscillatory behaviour making $T_2 < T_1$. The lattice motion makes $T_0 > 10$ K.

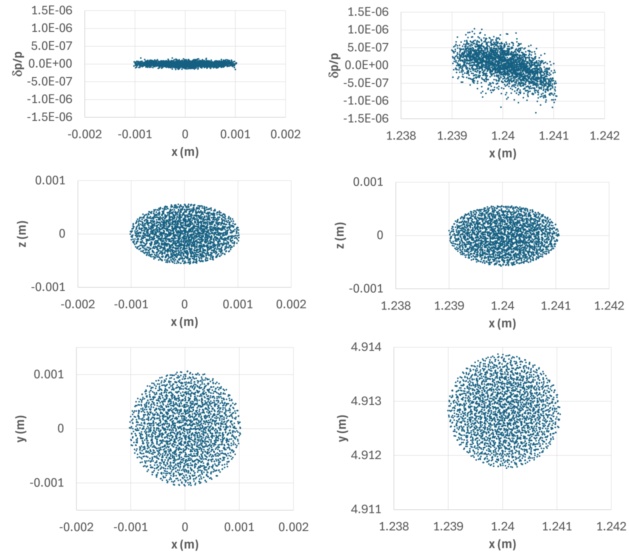

Figure 3: Particle distribution at injection (left) and t = 63.3 ms (right).

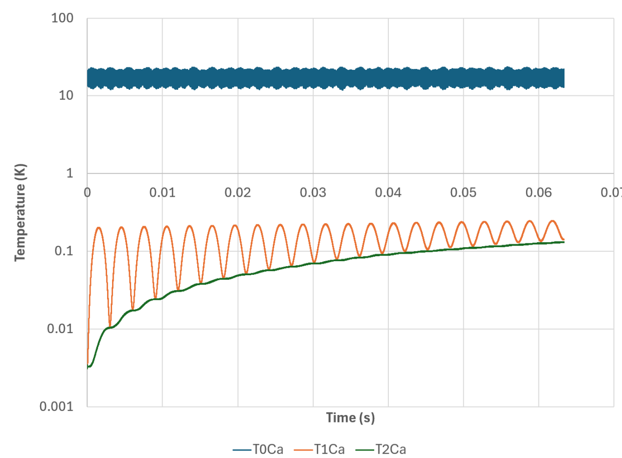

Figure 4: Bunch temperatures $T_{0,1,2}$ over time.

Changing the shape of the bunch with different ion trap parameters reduced the heating rate further, as shown in Figure 5 using a vertically taller bunch. This shape may make it less sensitive to dispersive effects in the horizontal plane. A heating rate of 0.8353 K/s was observed over 5 ms.

First Order Stability Analysis

In some cases, when the oscillation field profile $b(r)$ was changed from that in the previous section, velocity distribution parameters (and the temperature) showed exponential growth over ~100 turn time scales.

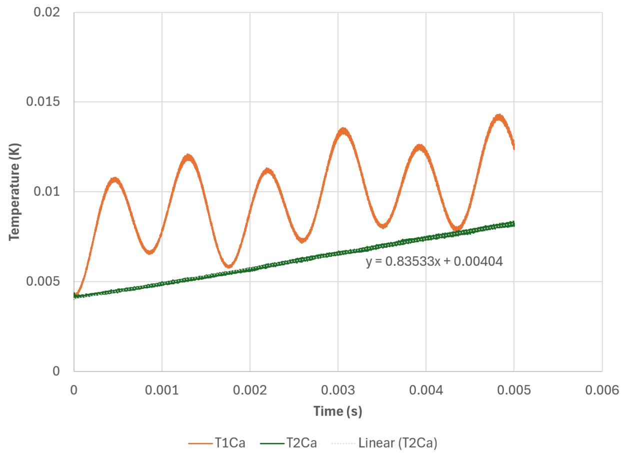

Figure 5: Reduced heating rate achieved with $\sigma_{x,y} = 229\ \mu$m, $\sigma_z = 432\ \mu$m bunch.

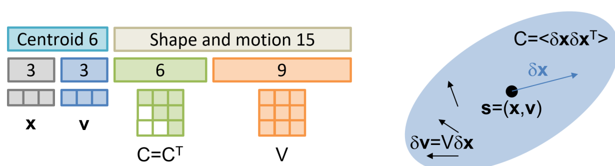

Figure 6: Parameters of the ellipsoid bunch model.

To search for collective instability modes, the model in Figure 6 was used. Here, the bunch is approximated by a uniform ellipsoid of charge with zero temperature. It is parameterised by its centroid $\mathbf{s} = \langle(\mathbf{x}, \mathbf{v})\rangle$, covariance matrix $C = \langle \delta\mathbf{x}\delta\mathbf{x}^T \rangle$ and velocity distribution V where $\delta\mathbf{v} = V\delta\mathbf{x}$. The centroid $\mathbf{s}$ evolves as a single particle while the 15 parameters (C, V) have time derivatives calculated in [10]. This captures the dynamics spatially to first order, so higher-order modes and heating of phonon motion are not included.

The oscillation field was changed by using $n_c = 3$ terms and defining multipole strength multipliers $D = b(R)/b_s(R)$, $Q = b'(R)/b'_s(R)$ and $S = b''(R)/b''_s(R)$ where $b_s(r)$ is the stable oscillation field used in the previous section. Combinations of $c_{0,1,2}$ can be found to produce any desired (D, Q, S).

For each (D, Q, S) setting, matched values of (C, V) were found that are unchanged after one turn. The Jacobian $\frac{\partial(C,V)_{out}}{\partial(C,V)_{in}}$ is a 15×15 matrix showing the effect of small perturbations to the bunch shape or velocity distribution. If it has an eigenvalue with modulus greater than one, there is an instability with growth rate given by the logarithm of that modulus. The effect of scanning D and Q on stability is shown in Figures 7 and 8. The previous stable solution is shown by a star at $(1, 1, 1)$ and lies between two unstable zones labelled in Figure 7.

Figure 9 shows the full set of eigenvectors for the Jacobian matrix of the stable solution. Seven pairs with complex eigenvalues are written as their two real combinations, which span the plane that periodic oscillations take place in. Zone 1 corresponds to eigenvector pairs P5 and P6 becoming unstable, which involve velocity in the horizontal plane. Zone 2

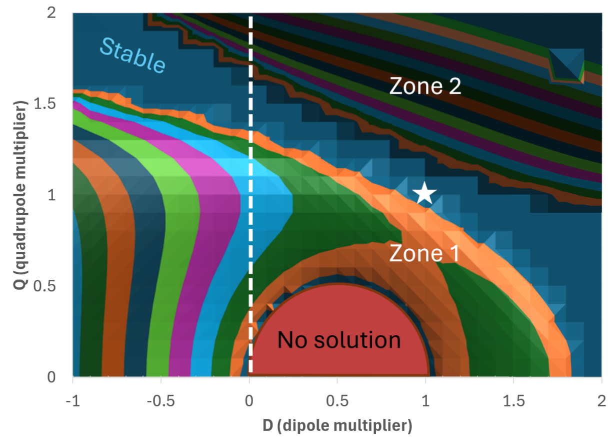

Figure 7: Growth rate in the first order model as D and Q are varied, $S = 1$. Colour contours are every 0.002 turns^{-1}.

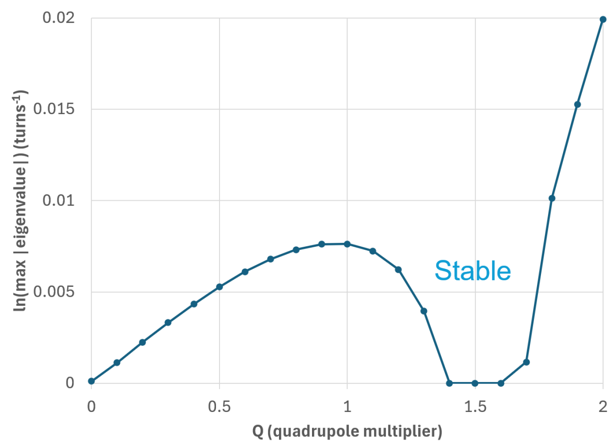

Figure 8: Model growth rate as Q is varied, $D = 0$, $S = 1$.

	Cxx	Cxy	Cxz	Cyy	Cyz	Czz	Vxx	Vxy	Vxz	Vyx	Vyy	Vyz	Vzx	Vzy	Vzz
P1	0.00	0.00	0.00	0.00	0.00	0.66	0.00	-0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.75
	0.00	0.00	0.00	0.00	0.00	-0.60	0.00	0.01	0.00	-0.01	0.00	0.00	0.00	0.00	0.80
P2	0.00	0.00	0.86	0.00	0.24	0.00	0.00	0.00	-0.01	0.00	0.00	0.02	-0.12	0.43	0.00
	0.00	0.00	0.24	0.00	-0.86	0.00	0.00	0.00	0.02	0.00	0.00	0.01	0.43	0.12	0.00
P3	0.00	0.00	-0.87	0.00	0.19	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.10	0.44	0.00
	0.00	0.00	-0.19	0.00	-0.87	0.00	0.00	0.00	0.02	0.00	0.00	0.00	-0.44	0.10	0.00
P4	0.53	0.67	0.00	-0.52	0.00	0.00	-0.01	0.01	0.00	0.01	0.01	0.00	0.00	0.00	0.00
	0.62	-0.49	0.00	-0.61	0.00	0.00	0.01	0.01	0.00	0.01	-0.01	0.00	0.00	0.00	0.00
P5	0.06	-0.02	0.00	0.04	0.00	0.00	0.53	0.21	0.00	-0.29	0.76	0.00	0.00	0.00	0.00
	-0.15	-0.01	0.00	-0.10	0.00	0.00	0.20	-0.52	0.00	0.75	0.29	0.00	0.00	0.00	0.00
P6	-0.02	0.13	0.00	0.01	0.00	0.00	0.56	0.08	0.00	0.11	-0.81	0.00	0.00	0.00	0.00
	0.15	0.02	0.00	-0.11	0.00	0.00	0.08	-0.56	0.00	-0.80	-0.11	0.00	0.00	0.00	0.00
P7	0.00	0.00	0.03	0.00	0.01	0.00	0.00	0.00	0.30	0.00	0.00	-0.95	0.00	0.00	0.00
	0.00	0.00	0.01	0.00	-0.03	0.00	0.00	0.00	-0.97	0.00	0.00	-0.24	0.00	0.00	0.00
S	-0.70	0.00	0.00	-0.69	0.00	0.19	0.00	0.01	0.00	-0.01	0.00	0.00	0.00	0.00	0.00

Figure 9: Stable eigenvectors for $D = Q = S = 1$.

corresponds to pair P7 becoming unstable, involving velocity coupling between the horizontal and vertical planes.

For the stable solution, the seven eigenvalue pairs $e^{\pm 2\pi i f}$ have $f = \frac{1}{13.9}, \frac{1}{19.6}, \frac{1}{21.4}, \frac{1}{179}, \frac{1}{485}, \frac{1}{709}, \frac{1}{958}$ turns^{-1} for P1 through P7 respectively. The remaining single eigenvector S has eigenvalue 1 and comes about because the model has an odd number of dimensions (15). This unit eigenvalue allows multiple matched solutions where the vertical bunch aspect ratio $\sigma_z/\sqrt{\sigma_x\sigma_y}$ can be freely chosen within a range.

REFERENCES

[1] H. J. Metcalf and P. van der Straten, "Laser Cooling and Trapping of Neutral Atoms", in The Optics Encyclopedia, Vol. 38, pp.847–853 (Wiley-VCH Verlag GmbH & Co. KGaA, Weinheim, Germany, 2007).

[2] R. Takai, H. Enokizono, K. Ito, Y. Mizuno, K. Okabe and H. Okamoto, "Development of a Compact Plasma Trap for Experimental Beam Physics", *Japan. J. Appl. Phys.* **45**, No. 6A pp.5332–5343 (2006).

[3] K. Izawa, K. Ito, H. Higaki and H. Okamoto, "Controlled Extraction of Ultracold Ions from a Linear Paul Trap for Nanobeam Production", *J. Phys. Soc. Japan* Vol. **79**, No. 12, p.124502 (2010).

[4] S. L. Sheehy, E. J. Carr, L. K. Martin, K. Budzik, D. J. Kelliher, S. Machida and C. R. Prior, "Commissioning and First Results of the IBEX Linear Paul Trap", in *Proc. IPAC 2017*, Copenhagen, Denmark, May 2017, pp.4481–4484. doi:10.18429/JACoW-IPAC2017-THPVA027

[5] L. K. Martin, S. Machida, D. J. Kelliher and S. L. Sheehy, "A study of coherent and incoherent resonances in high intensity beams using a linear Paul trap", New J. Phys. **21** 053023 (2019).

[6] S. J. Brooks, "Extraction of Coulomb Crystals with Limited Emittance Growth", in *Proc. IPAC 2024*, Nashville, TN, May 2024, pp.460–463. doi:10.18429/JACoW-IPAC2024-MOPR04

[7] S. J. Brooks, "Ultra-Low Emittance Bunches from Laser Cooled Ion Traps for Intense Focal Points", in *Proc. HB2023*, Geneva, Switzerland, Oct. 2023. doi:10.18429/JACoW-HB2023-TUC3I1

[8] H. Okamoto, H. Sugimoto and Y. Yuri, "Coulomb Coupling and Heating of Charged Particle Beams in the Presence of Dispersion", J. Plasma Fusion Res. SERIES, Vol. **8**, pp.950–954 (2009).

[9] S. J. Brooks, "Maxwellian Fields for a Ring derived from Multipole End Fields", note BNL-227922-2025-TECH, available from https://www.osti.gov/biblio/2530893 (2024). doi:10.2172/2530893

[10] S. J. Brooks, "First Order Transport of a Cold Uniform Ellipsoid of Charge", note BNL-229092-2025-TECH, available from https://www.osti.gov/biblio/3000537 (2025). doi:10.2172/3000537